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DAMAGE ASSESSMENT SYSTEMS AND THEIR RELATIONSHIP TO POST-NUCLEAR-ATTACK DAMAGE AND RECOVERY*

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INTRODUCTION

PURPOSE

The purpose of this paper is to present some thoughts on the problems of post-nuclear-attack disaster in terms of damage assessment or analysis systems and the relationship of damage assessment or analysis systems to postattack recovery. This paper will attempt to consider the problems more specifically associated with, and the plans for, damage assessment or analysis; and, to relate the potential solutions or plans to the probable success of United States recovery from a mass nuclear weapons attack.

The term "damage assessment system(s)" will be used hereinafter even though there is a trend toward the use of other terms such as "damage analysis" and "resource analysis" system.

FUNDAMENTAL PROBLEM AND ITS BACKGROUND

The underlying problem in considering nuclear damage assessment and recovery is that there is but little precedent or experience in the United States upon which to draw in devising solutions.[1,2] It is true, of course, that the results of the atomic bombing of Hiroshima and Nagasaki have been thoroughly studied and analyzed, but the results of such studies and analyses are not of direct benefit[3] when viewed in the light of the present or the future for the following reasons:

a. The targets (Nagasaki and Hiroshima) were totally unprepared for atomic attack. In addition there was little scientific knowledge and no public knowledge in Japan of the effects of nuclear weapons at that time.

b. Japan was already a beaten belligerent, and thus incentive to rapid recovery from the results of the attacks was lacking.

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†This paper originally appeared as a thesis by Colonel Ramsey when he was a student at the Industrial College of the Armed Forces, Washington, D. C. (Residence Course 1956-1957).

[1]U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 1.

[2]Congress, House Subcommittee of the Committee on Government Operations, Civil Defense for National Survival.

[3]Spear, Ralph, Civil Defense Planning, p. 3.

c. The two targets were essentially isolated. There was no "mass" in the attacks in the sense it is understood now; and the weapons used were, by today's standards, of relatively low yield.

d. Scientific and public knowledge in the United States on the effect of nuclear weapons has advanced to the point where there is little comparison between these factors as they currently exist in the United States as against the total lack in Japan at that time.[4]

Since the bombings at Hiroshima and Nagasaki, there have been an extensive series of tests in Nevada and in the Pacific and Bikini and Eniwetok. These tests have yielded vast amounts of useful data which are relevant to the problems of nuclear damage assessment and recovery.[5] However, such data, for the most part, are useful primarily as a point from which estimates, calculations, or extrapolations can be made when applying them to the situation which might occur in the United States if subjected to a massive nuclear attack upon a large number of its centers of population, Government, and industry; and upon strategic air, naval, and other key military installations. There is no direct precedent or experience upon which to draw for solutions to problems for the simple reason that no such event has, fortunately, ever struck this Nation.[6,7]

BACKGROUND

The question of nuclear damage assessment has, until relatively recently, received somewhat less emphasis than many other questions more directly related to recovery from the damage which might result from a nuclear attack. It is true that the Atomic Energy Commission (AEC) together with the Armed Forces Special Weapons Project (AFSWP) has given the question a considerable amount of attention; and latterly, the Department of Defense (DOD), the Office of Defense Mobilization (ODM), and the Federal Civil Defense Administration (FCDA) have applied much effort upon the question.

In the case of the AEC and AFSWP, effort has been and is expended primarily on the determination of the basic parameters relating to the effects of nuclear explosions upon various kinds of structures and things; and, upon various personnel situations, together with the effects of meteorological and other conditions upon these parameters. The FCDA, on the other hand, has made wide use of the AEC data with a view to determining methods of estimating the probable effect of such explosions on urban targets. This effort on the part of the FCDA resulted in, among other things, the publishing of a manual, TM-8-1.[8] Meanwhile the U. S. Air Force made studies of the question from the point of view of damage assessment. These studies resulted in the publishing of a report entitled the "Bomb Damage Problem."[9] This Air Force study and its report represented only a proposal outlining a damage assessment system and was not carried to the execution stage by the Air Force.

The ODM, too, has an interest in the question through its responsibilities for matters pertaining to mobilization planning, maintenance of the mobilization base and potential, together with related operational functions and responsibilities following an attack. This agency is very actively pursuing solutions to the problem at this time.

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[4]U. S. Federal Civil Defense Administration, 1953 Annual Report, p. 4.
[5]U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 1.
[6]Ibid., p. 1.
[7]U. S. Congress, House Subcommittee on Government Operations, op. cit., p. 1185.
[8]U. S. Federal Civil Defense Administration, Civil Defense Urban Analysis.
[9]U. S. Air Force, Office of Deputy Chief of Staff Comptroller, Directorate of Management Analysis, Bomb Damage Problem, 5 Feb 1954, 22 p.

The DOD is likewise actively engaged in a program relating to damage assessment. This program grew out of the necessity to evolve a bomb-damage assessment system which would be responsive to the needs of the military. The interest from this point of view is related to the need of the military to determine the effects of an attack upon the various kinds of bases and logistical installations. Such information is vitally essential to making an estimate of the Nation's military capability to fight the war following an attack.

Thus it is seen from this background that the question of bomb damage assessment has assumed key importance in postattack recovery; and that as a result of the relatively new interest in the question, considerable effort is being applied by the DOD, ODM, and FCDA on developing satisfactory damage assessment capabilities.

NEW EMPHASIS

New emphasis was given to the problems of bomb-damage assessment upon the announcement by Premier Malenkov of Russia to the Presidium of the Supreme Soviets on 8 August 1953 that: "The Government of the Soviet Union must inform the Supreme Soviet that the United States no longer possess a monopoly of the hydrogen bomb."[10] This emphasis was further heightened when the implications of the probable results of radioactive fallout became more keenly appreciated.[11] In a word the achievement by the Russians of a thermonuclear capability focused new attention upon bomb-damage assessment problems as did no other single event since World War II.

BOMB-DAMAGE ASSESSMENT AS A TOOL

Bomb-damage assessment is not in itself an end. It is, of course, highly important to certain departments and agencies of the U. S. Government, as well as State and municipal governments, that they be able by some means or other to determine the results of a nuclear attack. Such a determination should be timely and accurate. A bomb-damage assessment system is thus a genuinely needed tool serving the end of providing a computation of the results of an attack.

Basically, the necessity for a damage-assessment system stems from the fact that Russia's nuclear capability is now considered to be of a very large scope. If Russia chose to make an attack suddenly and in mass on the United States, the resulting possible damage would be so great that it is, in effect, necessary to quickly and accurately determine the degree of "vastness" of the damage. Such knowledge is necessary in order that certain fundamental questions can be answered and decisions made even under the stresses of the moment and within hours following the attack. Such fundamental questions might include some such as these:

a. To what extent can the surviving economy function to prosecute the war?

b. Is the morale of the people such that there is a will to continue the fight on the one hand, and to go beyond the bare actions to survive on the other?[12]

c. How shall survival and recovery plans be executed?

d. What will be the nature of military action following the exchange of nuclear blows?

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[10] Project East River, Review Committee, 1955, 1955 Review of Project, p. 1.

[11] U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 1.

[12] U. S. Congress, House Subcommittee of the Committee on Government Operations, op. cit., p. 1039.

As a matter of fact, the circumstances following an attack could conceivably reduce the problem of answering these questions to a case of choosing, on the basis of "either or else," question a or c. Obviously, rapid and accurate damage assessment is a key to assisting in arriving at answers to these most difficult questions, or as indicated, in extremis, the making of a choice of fighting or just surviving.[13]

PROBLEM AND ITS KEY

PROBLEM

For the purposes of this paper, the problem is being considered in terms of an attempt to analyze and evaluate the efficacy of presently available and prospective damage-assessment systems; and to relate in an objective fashion, the damage-assessment systems to postattack recovery.

ASSUMPTIONS

In order to provide a basis upon which an analysis and evaluation of the problem can be rested, the following assumptions are postulated:

a. That, if the enemy chooses to attack, he will avail himself of the advantages of both strategic and tactical surprise to the greatest extent that circumstances will permit.

b. The enemy will employ the principle of "mass." In this case mass is assumed to include maximum delivery of nuclear weapons on the largest number of key United States targets.

c. The enemy will select the optimum weapon, or weapons, for employment on each of the targets with due consideration to the lethal effectiveness of radioactive fallout resulting from surface or near-surface bursts of thermonuclear weapons.

d. The traditional United States distaste for taking the role of an aggressor will result in a situation wherein the enemy will strike first regardless of his ability to actually achieve either strategic or tactical surprise. This therefore assumes his success with respect to assumptions a, b, and c.[14]

e. In spite of the devastation that is likely to result, the American people and the surviving, or newly rising, leaders will have the spirit, morale, and will which will carry them to efforts above and beyond the mere concern for personal survival.

f. The United States will retaliate in kind and will follow up such retaliation with a maximum war effort commensurate with the means available and with a will to gain ultimate victory.

NATURE OF THE BOMB-DAMAGE ASSESSMENT PROBLEM

Until 1953 the question of bomb-damage assessment was largely a matter of applying certain techniques and data to the circumstances and conditions which were peculiar to each of the potential targets should they fall under attack by weapons in the kiloton range of sizes. It was generally assumed that the enemy would employ air bursts at optimum altitude in order to achieve maximum blast and thermal effects from atomic bombs. While a mass atomic bomb

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[13] Ibid.

[14] U. S. Congress, House Committee on Government Operations, Civil Defense for National Survival, p. 1 and 15.

attack under these conditions could be expected to cause very considerable damage and relatively heavy casualties, these effects are utterly paled by the destruction and casualties which can be anticipated from the use by the enemy of thermonuclear weapons bursting at or near the surface.[15] Much work has been accomplished and published for the A-bomb situation.[16,17]

Civil defense agencies, even at the municipal level, could and did to a certain extent, apply the techniques and data available to them to estimates of the probable damage that would result from an A-bomb attack. Such computations were relatively simple manual operations. The results obtained from these methods were of sufficient validity that it was possible to base various kinds of civil defense plans upon them. In fact the FCDA based certain principles and policies in its earlier years of effort on the results of such computations. For example, the FCDA based its principle of "duck and cover" on such methods of assessment.[18] However, when somewhat later the probability of obtaining early warning improved, the question of evacuation became a principle of the FCDA's planning. Since 1953 and in particular in 1954, the FCDA has reassessed this principle and is now considering both evacuation and shelter.[19]

Mass shelter is being more and more considered as the ultimate solution, however, due to the uncertainties of the reliability of warning systems, the inherently short time available for warning with intercontinental ballistic missiles (ICBM), and, in particular, the necessity for providing protection to the population from fallout.[20,21] At the present time the question of shelter vs. evacuation is very much a matter of controversy and the resolution of the controversy is not within the purview of this paper.[22]

At any rate, originally, the question of bomb damage assessment was a relatively simple one; and while important in the A-bomb situation, it was of somewhat less importance prior to the advent of the H-bomb.

In 1954, when the potential magnitude of the combined effects of a large-scale thermonuclear attack became genuinely palpable, the question of an accurate and rapid method of damage assessment assumed major importance. The U. S. Air Force, as mentioned, had prepared a proposal[23] which was taken up by the FCDA. The FCDA followed the principles within the USAF proposal in a project which came into being through a contract let by the FCDA to the Stanford Research Institute (SRI).[24,25]

The reasons for the importance of bomb-damage assessment stem from three primary areas of interest: military defense, civil defense, and war mobilization.

In the area of military defense, the necessity for accurate, rapid bomb-damage assessment is more or less self-evident. It stems from an urgent need to determine at a very early hour the results of a nuclear attack on air bases, naval bases, operational forces, and on the various military supply and logistical installations. Such assessment is patently required to

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[15] Life Insurance Association of America, Committee on Civil Defense, General Aspects of Civil Defense, p. 39.
[16] U. S. Federal Civil Defense Administration, Civil Defense Urban Analysis.
[17] U. S. Los Alamos Scientific Laboratory, Effects of Atomic Weapons.
[18] U. S. Federal Civil Defense Administration, 1953 Annual Report, p. 5.
[19] U. S. Federal Civil Defense Administration, 1954 Annual Report, p. 8 and 9.
[20] U. S. Congress, House Subcommittee of the Committee on Government Operations, op. cit., p. 1185.
[21] Ibid., p. 1178-1179.
[22] U. S. Congress, House Committee on Government Operations, op. cit., p. 2.
[23] U. S. Air Force, op. cit.
[24] U. S. Federal Civil Defense Administration, 1954 Annual Report, p. 30.
[25] Stanford Research Institute, Summary Report for Civil Defense Damage Assessment Program, 1956.

permit an evaluation of the strength of the "forces in being" following such an attack. The determination of the remaining strength of these forces, together with the status of their materiel and logistic situation, is germane to a determination of the nature of the war which can be fought in the phases following the exchange of massive nuclear blows.[26]

For civil defense the problem has become one of a potential disaster of such magnitude, in terms of physical damage, casualties, and the effects on human morale, that the problem can no longer be left to solution at the local municipal level. It is now necessary to consider bomb-damage assessment and casualties on a metropolitan area basis[27,28,29] rather than on a single city basis. Furthermore, local self-help with the possible deployment of assistance from outside the disaster area is a principle which appears to be no longer valid.

The FCDA has adopted the principle that assistance to attacked communities must come from outside the local disaster areas and most probably from great distances.[30,31] The metropolitan area concept encompasses coordination problems that involve city, county, and State boundaries. The "outside assistance" effort means in effect that when the scope of the damage becomes known, the entire available resources of the Nation must be responsive to the needs of survival and recovery of the attacked areas. Fallout patterns must be measured and computed for each ground or near-ground burst and made available to areas which are threatened by their spread.[32]

These are the highlights of the importance and the nature of the problem of bomb-damage assessment in the era of the H-bomb and its relationship to the responsibilities of the FCDA.

But bomb-damage assessment is of vital importance to war mobilization also. The United States strategy calls for immediate mass retaliation against the enemy. This is to be followed by whatever military action can be prosecuted which will result in ultimate victory for the United States over Russia. As it now stands, such action will have to be undertaken by the forces in being at the completion of the nuclear exchange, plus such mobilization as the Nation can undertake with whatever mobilization reserves might exist immediately following the exchange of nuclear blows. The ability, or even the necessity, to continue the fight following the initial nuclear phase is an imponderable of no little consequence to the U. S. Government.

One can only assume that Russia will have the will to continue the fight within its capabilities following retaliatory nuclear strikes. This assumption stems from the fact that, barring the possibility that the initial Russian decision to make the attack was based on other than whimsey,[33] there were sufficiently strong reasons for the Russian masters to believe that they could attack the United States in that fashion with every reasonable expectation of winning the war. Further, that their success would include the acquisition of a defeated United States that would not be a total economic liability. This being the case, it will be essential that the

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[26] U. S. Department of Defense, Office of the Assistant Secretary of Defense (S&L), 1956-57, OSD.ADNA.CUS. (C)

[27] U. S. Congress, House Subcommittee of the Committee on Government Operations, op. cit., p. 1186.

[28] U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 2.

[29] Project East River, op. cit., p. 10.

[30] U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 17.

[31] Project East River, op. cit., p. 10.

[32] U. S. Federal Civil Defense Administration, 1955 Annual Report, p. 17.

[33] U. S. Congress, House Committee on Government Operations, op. cit., p. 14.

President and the National Security Council (NSC) be in possession of sufficient information regarding the capability of the Nation not only to mobilize for war but sufficient information to determine, under extreme circumstances, what might be termed the "tempo" of the fight with the forces in being following the initial phases of the war.[34] Furthermore, the results of the attack upon the population and the economy of the country could conceivably be of such vast magnitude[35] that the "either or else" decision referred to on page 202 might have to be made by the President.

In any event, and without going to extremes, it is clear that the magnitude of the effects of a massive nuclear attack will require major decisions on the part of the President, his Cabinet, and his other advisers. Such decisions may involve, as examples, such things as:

a. The temporary employment of the military for civil defense.

b. The emergency allocation of all types of transportation between the needs of civil defense and of the military.

c. Even a decision as to how much, if any, of the remaining economic potential of the country can be diverted to survival and recovery at home vs. how much can be made available for mobilization or even for the prosecution of military operations against the enemy with the forces in being.

One need only to study the current plans of the ODM[36] to visualize the scope and complexity of the problems and decisions which will come about as the Nation digs out from under.

These kinds of decisions can only be based on timely, accurate damage assessment data.

To provide a means leading to the answers to the questions and problems such as those previously enumerated, and to provide essential information upon which plans can be either executed or altered for execution, bomb-damage assessment clearly becomes the key tool leading to the solutions required.

PRINCIPAL FEATURES OF CURRENT AND FUTURE BOMB-DAMAGE ASSESSMENT SYSTEMS

Initially, as mentioned on page 202, under assumptions, bomb-damage assessment systems were based on rather simple computations. The process was accomplished either manually or with ordinary office type calculating machines together with the use of bomb-damage templates and other similar aids. These methods were tedious and time consuming and were therefore primarily of use only for local civil defense purposes.

But with the new importance of obtaining timely, accurate, and comprehensive information of what might actually have happened to the Nation following a massive nuclear attack, a more sophisticated approach was clearly indicated. The use of electronic computers is the obvious tool by which the necessary solutions can be generated.[37] As a result of this, there are currently two electronic bomb-damage assessment systems in existence to a greater or lesser degree, with a third (that of the DOD) a very likely probability.

The first of these systems is that of the FCDA. This system is of particular interest because its development required certain basic decisions to be made which have resulted in a

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[34] U. S. Federal Civil Defense Administration, A Progress Report on The Damage Analysis System, p. 1.

[35] U. S. Congress, House Subcommittee of the Committee on Government Operations, op. cit., p. 1180-1181.

[36] U. S. Office of Defense Mobilization, ODM MPD-M. (C)

[37] U. S. Congress, House Subcommittee of the Committee on Government Operations, op. cit., p. 1040-1041.

most significant degree of standardization of certain parameters and input data. This standardization has resulted in making the other systems compatible with that of the FCDA. This permits a large measure of "insurance" plus the ability to share the workload and to allow the exchange of data between the FCDA's and the other systems. The basic principle of the FCDA's system is described in sketchy fashion in the Department of the Air Force pamphlet previously referred to.[38]

The SRI, in reporting the results of its work for the FCDA, has prepared a comprehensive description of the basic derivation of the FCDA system.[39] The FCDA, meanwhile, has very recently published a new, brief descriptive pamphlet which outlines the system and its application to civil defense situations.[40] In view of these sources of information, this paper will not attempt to describe in detail the technical features of damage-assessment systems. However, some comments regarding certain of these details will be covered only for the reason that they seem pertinent to the purpose of the paper and are common to the other systems. These will be touched on in the following paragraphs.

There are two general classes of input data required by any electronic bomb-damage assessment system:[41]

a. Those which can be determined and stored in the memory elements of the computer prior to the attack (prestrike data).

b. Those which can be determined only as a result of and subsequent to the attack (poststrike data).

In the first category, the range of kinds of prestrike data is most extensive and is related to the whole range of responsibilities and areas of interest of all the various departments and agencies of the Government which are concerned with the results of an attack. But of all the kinds of preattack input data there are, certain kinds can be considered basic. Among these are:

a. Population—its size; its distribution in locations; the variations of its distribution between daytime and nighttime; and certain data regarding special classes of population such as medical, nursing, public safety personnel, and so on.

b. The nature and location of installations—such as, for example, factories, transportation networks, food storage and processing activities, utilities, military installations, and so on through the entire range of physical resources. This includes certain special things which might tend to reduce damage or assist in the recovery from the attack such as terrain feature or manmade installations which can act as fire-breaks or prevent the buildup of fire storms, or reduce the hazards from fallout or other manifestations of nuclear explosions.

The matter of location, which, from the preceding, is obviously of key importance to preattack input data, required certain basic standardization agreements before the damage assessment program could proceed. The final solutions which have been adopted for use as standard by all systems are:

a. All locations will be referred to a grid system based on the Universal Transverse Mercator (UTM) grid.[42,43] This grid system was adopted primarily because there were no

[38] U. S. Air Force, op. cit.
[39] Stanford Research Institute, op. cit.
[40] U. S. Federal Civil Defense Administration, A Progress Report on the Damage Analysis System, p. 1-17.
[41] Stanford Research Institute, op. cit., p. 4.
[42] U. S. Department of the Army and of the Air Force, Universal Grid Systems, (TM-241).
[43] Stanford Research Institute, op. cit., p. 5-7.

other generally adaptable simple systems; and, because the use of latitudes and longitudes is complicated and cumbersome. Furthermore, the UTM grid system can be applied to any map which is based on the Mercator projection and this type of map projection is common for most maps. Not the least of the reasons for adopting the UTM grid system was also the fact that this is the system which is the standard for the military. Since the UTM grid system is a method of expressing locations on a map in terms of a standard arrangement of numerical digits, it also has the very obvious advantage of simplicity in programing computers or in otherwise being inserted into computer systems either on punched cards or on tapes.

b. Population location data are standardized in terms of the National Location Code.[44] This code is based on the employment of the Bureau of Census measures of population information in terms of what are known as "standard locations" and "census tracts" for urban areas, and an arbitrary standardized measure for rural areas wherein all the population of a county is considered to be located in a single town. By the adoption of population location data in an already standardized system such as that of the Bureau of the Census, all the mass of that Bureau's population and other kinds of data are then readily available and immediately adaptable to use in compiling preattack input to the various damage-assessment systems. The Bureau of Census data do, however, have the disadvantage of being frequently not up to date.

By the standardization of the foregoing basic kinds of preattack inputs so that they are fully compatible in any system, it is apparent then that all sorts of preattack input information relating to people and to "things" can be programed into a computer. Being standardized and based on information that either already exists or which can be readily determined in advance of an attack permits the full utilization of the "memory" capability of electronic computers.

Now, as previously mentioned, the range of kinds of preattack input data is determined by the interests and responsibilities of all the departments and agencies of the Government which have interests or responsibilities in the event of an attack.[45] This means that, potentially, a damage assessment system can produce a vast mass of information ranging from what might be described as an overall "big picture" down to a great amount of detail regarding the status of one certain, particular item of concern to an operating office of some agency or department—provided only that the necessary input data on that particular item had been set up for entry into the system. The limited availability of funds, personnel, and time are current limitations on developing the full potential scope of preattack input data.

As indicated previously, there are, or there will be by midsummer 1957, two damage-assessment systems in operation. These two systems have variations in their potential to assimilate and thence to produce bomb-damage data. These are the systems sponsored by the ODM and by the FCDA. There is also a distinct possibility that the DOD will also place a system in operation in the foreseeable future. The system of the FCDA is based on an IBM installation. It is classed as an intermediate speed system and is thus somewhat limited in the rapidity with which it can perform its overall function. It is nonetheless a most useful and practical system. It has certain advantages with regard to the level of skill and training of personnel required to program and otherwise operate the equipment.

On the other hand, the system of the ODM is to be a "high speed" system based on the UNIVAC. While this type of a system requires some personnel having highly specialized skills, its capabilities are vastly greater than that of the FCDA's IBM system.

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[44] Ibid.
[45] Ibid., p. 6-7.

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The FCDA will have its system located at its dispersed location while that of the ODM will be located at that agency's dispersed location which is different from that of the FCDA. If and when the DOD establishes its system, it is expected that it will be located in still a third dispersed location. Since much of the preattack input data is adaptable to and will be capable of being programed into all these systems, the plan of the ODM is to tie the systems together in their dispersed locations by an electronic data transmission system. Such an electronic data transmission system will then permit flexibility within the overall damage assessment capability, provide an "insurance" factor in that if one or two systems are inoperative, the remaining system could assimilate all the input data of the others and thus compute the desired results; and finally, such interconnection by electronic data transmission systems will permit each of the three systems to make assessments in a manner which is most directly of interest to the fields of primary interest of the sponsoring agency. Of this point there will be further discussion later.

Part of the electronic data transmission system is already in existence between the dispersal locations of the ODM and the FCDA.

At this point it seems appropriate to note that there appears to be duplication in damage-assessment systems with the existence of two systems at this time and with prospects of a third in the future. It is also interesting to note that there is currently no overall written plan or guidance governing the development of the system.[46] The ODM has, however, a project under consideration which has the objective of preparing formal guidance and coordination of the various damage assessment systems. Meanwhile, coordination between the ODM, DOD, and FCDA is being effectively achieved by means of an ad hoc committee having membership from each of the three agencies and chaired by an ODM member.[47] It is the consensus among representatives of the three agencies concerned that while overlapping to a certain extent, the three systems are necessary for "insurance" against attack, as previously mentioned, and to provide for the three distinctly different general areas of interest of the three agencies.

This latter fact provides a frame of reference which is used to determine which of the systems will provide for the damage-assessment needs of the many various other departments and agencies of the Government. These other departments and agencies include, for example, the Department of Labor; Department of Agriculture; Department of the Interior; Department of Health, Education, and Welfare; and the Department of Commerce; the Interstate Commerce Commission; Civil Aeronautics Board; Civil Aeronautics Authority; and many others which have responsibilities or functions in a war and which functions and responsibilities might be affected by a nuclear attack.

In order for them to obtain their bomb-damage assessment data in terms of people or things in their respective areas of interest, each such aforementioned agency or department is required to reduce its particular items of interest into data which are in terms of the basic measures; Standard Locations converted to UTM coordinates plus certain other standardized descriptive information including vulnerability which permits its introduction as an input into a bomb-damage assessment system's computer. They are also required to define the nature of the bomb-damage assessment data which they desire to be produced by the system so that the programing personnel can program the computer accordingly.

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[46] Pettee, James, Dr., Office of Defense Mobilization, Interview, Wash., D. C., 9 Jan 1957.
[47] Ibid.

Finally, the damage-assessment data of these interested agencies can generally be categorized into two overall types:

a. That which is primarily related to civil defense functions, i.e., the question pertaining to survival, recovery, and rehabilitation.

b. That which is primarily related to the economic potential for war; or in other words, the area of overall interests of the ODM's mobilization and war controls functions.

There are many areas of overlap, but the division into these two categories provides the general guide by which the departments and agencies can determine which of the systems, other than the DOD's (if the latter is actually established), their preattack input data are to be programed for and from which their damage-assessment data are to be received. As a matter of fact the ODM and the FCDA delegations of authority to the various departments and agencies of the Government are indicative in a general way with the division of damage-assessment responsibilities as between the systems of the ODM and the FCDA.

At the risk of being criticized for oversimplifying the situation, further discussion of the matter of input data for the systems will not be made; but, attention will be directed to what is considered the weakest area in the whole concept of damage assessment utilizing electronic computers. This weak area is related to intelligence in the form of "postattack input data." (The term "poststrike" is used herein to mean the same thing.) Within the poststrike input data problem is the problem of handling fallout. The handling of fallout is, however, also complicated by the fact that it is also a consideration as preattack input data.

The range of postattack input data is considerably less than that of preattack input data. Poststrike data essential to the generation of damage assessment for any target are:[48]

a. The location of ground zero (GZ) in terms of UTM grid coordinates.

b. The height of burst; or whether surface, near surface, or subsurface.

c. Yield of the weapon.

d. Wind direction and velocity.

e. Status of the population, i.e., the time of the day and the distribution of the population, or degree of evacuation (if any), and the amount of warning (if any).

A discussion, which follows, of these poststrike input data reveals the reason for the foregoing opinion, i.e., that the area of poststrike input data represents a serious weakness in the present systems. Items a, b, and c of the list of poststrike input data all relate to the characteristics of the weapon and of its burst upon a target.

Location of Ground Zero

The first of these three pertaining to the location of GZ, at first blush would appear to be relatively simple to determine. But a little reflection on the subject will reveal that, as a matter of actual fact, the determination of the location of GZ after a nuclear attack, particularly one of a size in the megaton range, presents some genuine difficulties. In the first place, a reasonably precise determination of the location of GZ is required for assurance of a reasonably good assessment of damage to the target since certain of the results of a burst are related to distance.

The necessity of a reasonably precise determination becomes more important as the size of the weapon increases (except in the possible, but not very probable, event that the total

[48] U. S. Federal Civil Defense Administration, A Progress Report on the Damage Analysis System, p. 3.

damage area of the burst obliterates the entire target even if the weapon falls at the maximum limits of the circular area probable (CAP)). This is because it is what escapes the effects that we are interested in and not what suffered the effects. But at the present time no accurate, rapid means exist for the determining of the location of GZ in a manner which can be considered a practical "field" method. It is highly unlikely that survivors in the area, even if trained, would be capable of making even a reasonably accurate estimate of the location of GZ within a reasonably short time after the event.

It is probable that it would take several hours or even days before such observers, wardens, or other persons who survived could expose themselves to make a visual reconnaissance or observation which could be expected to develop sufficient information to make an estimate of the location of GZ. Furthermore, even if not pinned down by the effects of the burst, the state of mind of such survivors would probably not be conducive to an accurate determination. Firestorms, dust, smoke, and other such phenomena might well prevent visual observation of the blast area for an extended period of time after the explosion.

Finally, the question of communicating the information to appropriate centers for subsequent relay into the damage assessment centers poses a problem for such observers. It seems likely that ordinary communication means, other than special disaster circuits, would either be damaged or at best so overloaded that such observers would have difficulty in transmitting their information to an FCDA damage-control center which would then have to relay it onward. The subject of communications will be discussed in more detail later.

The only means of determining the location of GZ by other than the visual means just discussed that is a current capability is that of aerial photo reconnaissance. Such photographs, of course, will permit the location by aerial photo-interpretation techniques and thus represent a most useful method. Like the visual method, however, the ability to obtain sufficiently good photography will depend upon the weather, the winds, and whether or not firestorms resulted from the burst. Any and all of these effects can cause delays that may become very significant to obtaining a timely damage assessment from the systems. However, current plans are in existence which provide for the flying of such photo missions and aerial photo units have been designated to carry out this mission.[49] This method does have the advantage that the problem of communicating the results of such photo reconnaissance will be taken care of over military circuits. This is expected to be far more reliable and rapid due to the fact that there will be only a relatively few points from which the information will be introduced into the communication system; and, second, there is the inherent flexibility and reliability of the military communication networks.

Height of Burst

The second item on the list of prestrike input data, pertaining to height of burst, presents similar problems for any method relying on visual means. Additionally, this information cannot be obtained directly from aerial photo techniques. Therefore there is little present capability to determine this information and this is a contributing factor to the weakness in the systems.

Yield of the Weapon

The third item in the list, yield, cannot be determined by any direct means involving visual observation for the same reasons as previously stated. It can be deduced to a fair

[49] Wood, Marshall and Jensen, Office of the Assistant Secretary of Defense (S&L), Interview, Wash., D. C., 11 Jan and 1 Mar 1957.

degree of accuracy from such information as the size and the nature of the crater and of the "rings" of damage around it. Such deductions are likely to be slow in being arrived at and thus introduce a delay in rendering the final damage assessment.

Wind Direction and Velocity

On the list of poststrike input data, the fourth item can be more easily determined; or, if its determination at or immediately following the time of attack cannot for some reason be made, the data last available prior to the attack can often be of much value in making damage assessments. But, the part that wind direction and velocity plays in the matter of handling the assessment of radioactive fallout is a serious one and will be discussed later in connection with the discussion of the subject of fallout.

Status of the Population

The final item on the list of poststrike input data, status of the population and related matters, has a relationship to the controversial subject of shelter vs. evacuation. In either event the physical location of the population is patently germane to the computation of post-strike casualties. Furthermore, it has been possible by means of statistical analyses considering the particular target, the time of the day, the distribution of the population (daytime and nighttime), the amount of warning (if any), and similar factors, to make studies of the probable casualties for various targets. These techniques are based on the Bureau of Census data plus the studies of the SRI.

Certain of these data can be incorporated in the prestrike input so that time of burst and amount of warning could provide enough poststrike input to permit rather good postattack assessment of personnel casualties. Such assessment can be made more accurate and definitive, however, if the status of the population at the time of the attack can be introduced into the system as poststrike input data. The effect of fallout on population computations is something of an imponderable and a problem in itself. It will be discussed later.

The preceding discussion of the five essential items comprising the basic poststrike input data is indicated as being difficult of determination at the present time. Since this determination is, however, a key necessity to the calculation of bomb damage by the electronic systems, it is the opinion of the writer that the present limited capability in this area constitutes a palpable weakness in the systems. The question then follows: What measures are being taken to ameliorate this condition?

PROBLEM OF THE WEAKNESSES OF THE SYSTEMS AND THE PROSPECTS FOR SOLUTION

As previously mentioned, the five poststrike input items can be divided into categories: i.e., one relating to the burst of the weapon itself; one relating to weather conditions; and, one relating to the status of the population at the time of the burst. In considering the first—obviously there are more sophisticated methods of determining the location of GZ, the height of burst, and the yield of the weapon. It is a known fact that these facts are determined with considerable accuracy at the various weapons tests that are conducted at the proving grounds in Nevada and in the Pacific. Furthermore, the announcements in the press by the AEC of the conduct of nuclear tests by Russia implies that means for making such determinations do exist.

The fact that the techniques available at present are not available to damage-assessment systems is due to the following:[50]

a. Such methods are based on laboratory type techniques and therefore probably have limitations in their suitability to cope with multiple targets such as would characterize the kind of nuclear attack under consideration.

b. Such methods are of a nature which undoubtedly require specially trained technicians to convert the data into usable input data for a damage-assessment system.

c. Such methods are probably not adaptable to unattended operation at locations sufficiently remote from potential targets to be considered as safe.

d. Such methods are probably of a nature which would require the processing of the information obtained into some form that could be transmitted over a communication system. They are probably not, in other words, capable of transmitting their data automatically.

In the early period of civil defense planning, effort was devoted to the development of devices which might have utility in determining the location of GZ and the height of burst of kiloton range weapons. These devices included such simple things as the "flash dial," "lamp shade," "sphere," etc. These were passive devices of almost crude simplicity.[51] Their greatest advantage was in their low cost. Their chief disadvantages were that they required installations that were not only suitable for "seeing" the burst without themselves being destroyed, but the location had to be durable enough to remain in position itself. Wardens were required to know their location and would have to proceed to these locations to take readings after the blast. Not one of these simple devices was able to measure the yield directly. Obviously, they all suffer from the same disadvantages that visual reconnaissance does as discussed in a previous paragraph. None, of course, was adaptable automatic transmission of their data. Patently then, devices such as these have no real utility in the megaton situation.

But the fact that accurate, timely determination of poststrike input data is of such vital importance to the success of the various damage assessment systems, much effort is being placed on research to develop suitable methods of determining these data and then making them available to the systems.[52] Currently, the military have two projects under development[53] which have as their objective the achievement of a reliable, simple, unattended, economically feasible method of determining the location of GZ, the height of burst, and the yield of weapons dropped on most potential targets. The intent of these developments is to provide a means not only of determining these data, but also to transmit them in such a fashion as to be assimilable into any of the damage assessment systems. The fact that these projects are being undertaken by the military is due to two facts:[54,55,56]

a. The existence of military operational communication systems which are of such flexibility, reliability, and scope as to give the greatest assurance that the data can be transmitted.

b. The military has an urgent operational need for this kind of intelligence for its own purposes.

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[50] Ibid.
[51] U. S. Federal Civil Defense Administration, Civil Defense Urban Analysis, p. 48.
[52] U. S. Federal Civil Defense Administration, A Progress Report on the Damage Analysis System, p. 3.
[53] Wood and Jensen, op. cit.
[54] Ibid.
[55] Pettee, op. cit.
[56] U. S. Federal Civil Defense Administration, Interview, Wash., D. C.

In recognition of these factors, the ODM has delegated to the DOD the responsibility for providing for this portion of the damage-assessment input data for all the systems.

It is obvious, also, that the introduction of the five basic items of poststrike input data for each of the many potential targets (as many as perhaps 190 for future civil defense exercises have been considered) poses some very difficult problems in the field of communications even if it can be assumed that the problem of determining these data has been solved. No plans currently exist for any special communication arrangements to attempt to insure the availability of these data to any or all of the damage-assessment systems. One of the previously mentioned monitoring projects includes a planned capability, if feasible, to transmit the location of GZ, the height of burst, and the yield automatically in a code which is of a type compatible with IBM or UNIVAC inputs.

But the bare fact is that damage to the telecommunications network may well prove to be such as to cause delay, or even to deny the data to the damage-assessment systems. A real technical difficulty relating to any unattended monitoring device's ability to transmit its data over the telecommunication network under the conditions of likely damage is that some means must be devised to insure that the damage-assessment system can be advised of whether or not any particular target's monitoring devices can or cannot report. It would thus appear that communications of poststrike input data to the damage-assessment systems constitutes a weakness at present as well as in the foreseeable future.

PROBLEM OF FALLOUT

Fallout from megaton weapons bursting at or near ground level constitutes a formidable personnel casualty producing factor.[57] In fact any reasonable target pattern in which about 70 or more weapons in the megaton range whose total equivalent explosive effect is of the order of, say, 300 MT can be expected to almost completely blanket the continental United States with a high intensity radioactive fallout pattern in from 12 to 24 hours.[58] The casualty producing potential of such a fallout pattern could be expected to exceed that of the more direct effects of the various bursts unless large-scale shelter facilities are provided even in rural areas.

At the same time the determination of damage and casualties resulting from fallout presents an extremely difficult technical problem.[59,60] Basically, there are two factors which make for this difficulty. First, there is at present insufficient data on the question of how much radioactive material can result from weapons of different yields under various conditions of bursting, plus the behavior of the radioactive matter while suspended in the atmosphere; and, second, the behavior of the fallout with respect to varying meteorological conditions—particularly the effect of winds at the varying altitudes involved. The United States Weather Bureau has the task of solving this problem and is in fact working on it at this time. Meanwhile, the National Bureau of Standards (NBS) is also actively studying the problems relating to the adaption of electronic computers to the solving of the fallout problem in terms of the poststrike data, particularly, with regard to the handling of the winds.

Nevertheless, at this time, the effect of radioactive fallout in relationship to damage assessment is not yet within the capability of the electronic computers which form the basis of

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[57] U. S. Congress, House Committee on Government Operations, op. cit., p. 9-16.
[58] U. S. Federal Civil Defense Administration, Interview, Wash., D. C.
[59] U. S. Congress, House Committee on Government Operations, Civil Defense for National Survival, pt. 4, p. 756-762.
[60] U. S. Federal Civil Defense Administration, A Progress Report on the Damage Analysis System, p. 14.

the various damage assessment systems. As a matter of fact, except for a special computer developed by the NBS which can portray visually on a picture tube the pattern of fallout from an explosion on a single particular target, all computation of fallout patterns must be computed manually for each target. Thus the current limited ability to assess the effect of fallout represents a serious contributing factor to the weaknesses that currently exist in the damage-assessment systems.

SOLUTIONS TO THE DAMAGE-ASSESSMENT PROBLEM

GENERAL

In the preceding section, a problem was postulated and then followed by a discussion of the factors bearing on that problem. The problem was stated primarily in the form of a statement of the weaknesses that exist in the damage-assessment systems as they now exist. In the various parts of the discussion referred to, partial solutions were indicated in some instances, while in others, weaknesses lacking solution were stressed.

But the basic fact remains that the poststrike damage can, even at this time (or within the next very few months), be computed by one or both of two electronic computers. This is true provided, however, that certain poststrike input data can be fed into these systems accurately and rapidly; and, provided further, that these data can be communicated from their sources into the damage-assessment systems. Further, there is the proviso that the question of handling fallout by computer methods can be solved. In other words fully capable thermonuclear era damage-assessment systems are in existence but their genuine effectiveness following an attack hinges on solving certain technical problems and certain administrative problems as discussed in this section.

SOLUTIONS TO THE TECHNICAL PROBLEMS

As previously described, the technical problems are essentially all related to the question of poststrike input data. The solution to the technical problems obviously lies in increased research and development (R&D). Such R&D is in fact being prosecuted along lines discussed in this section. Lest it become necessary to assign a security classification to this paper, further details on the nature of this work and its prospects will be omitted.

SOLUTIONS TO THE ADMINISTRATIVE PROBLEMS

As mentioned on page 208, there is currently no overall formal guidance or plan governing the damage-assessment developments. An ad hoc committee provides informal coordination of the efforts to arrive at solutions to the technical problems that await the results of R&D for their solutions. This steering committee, due to the interest of its membership in the subject of damage assessment, is providing an informal course to guide the activity in the field of development leading to fully practical damage-assessment systems. The ad hoc steering committee has set as its goal the achievement of an integrated group of separate damage-assessment systems which is called the National Damage Assessment System.[61]

It is definitely to the credit of the departments and agencies concerned (DOD, ODM, and FCDA) that they have personnel associated with their respective programs and with the steering committee who are not only knowledgeable but also most enthusiastic and cooperative.

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[61] U. S. Federal Civil Defense Administration, Interview, Wash., D. C.

Thus despite the lack of formal plans and guidance, genuine, reasonably well-coordinated progress is being made. However, and purely as an opinion, some areas are not receiving adequate attention, nor has the informal "ad hoc" approach guaranteed a firm basis for insuring adequate funding responsibilities among the departments and agencies. While this question does not appear to have a really adverse effect on the progress of the programs of the sponsors to date, there is the definite possibility that such might occur in the future for the reason that in some instances, costs have been borne by indirect methods. This may well prove difficult in the future.

Under present circumstances it would appear logical that the Director of Defense Mobilization is the logical coordinator, on a formal basis, for the guidance and direction of further development of the damage-assessment systems so that they will in fact become the National Damage Assessment System. He should therefore develop a plan and provide the necessary policy guidance for the ODM, DOD, and the FCDA damage-assessment activities plus the necessary policy guidance for the many other interested departments and agencies of the U. S. Government. By thus formalizing the administrative matters connected with the development of the National Damage Assessment System, some of the "grey" areas associated with the problems of damage assessment—such as, one example, that of communications—which do not seem to be receiving sufficient consideration, would be more likely to receive the attention that they really require in order to insure that they are adequately developed and integrated into the system.

DAMAGE ASSESSMENT AND POSTATTACK RECOVERY

DAMAGE ASSESSMENT AND ITS RELATIONSHIP TO RECOVERY

As mentioned previously, damage-assessment systems are a key means in insuring the Nation's ability to cope with the results of a massive nuclear attack. Postattack recovery can be divided into three general areas of concern:

a. Military.
b. Mobilization and/or economic potential for war.
c. Civilian or "homefront."

MILITARY

The military's interest in the results of massive nuclear attack is, of course, to determine its strength in being since that strength is likely to be the Nation's warmaking capability for an indeterminate period following such an attack. Obviously, the military's immediate interest in damage assessment is to determine the results of the enemy attack on such installations as SAC bases, naval bases, CONAD bases, naval forces at sea, and the various supply and logistical installations and similar activities. The military is also vitally interested in the mobilization potential of the Nation, as well as whether or not, or to what extent, forces will become involved in assisting on the homefront. The latter considerations, however, are the primary interests of the ODM.

OFFICE OF DEFENSE MOBILIZATION

The ODM, which will have operational roles and probably a different organization and title,[62] is interested basically in the remaining resources of the Nation following the attack.

[62] U. S. Office of Defense Mobilization, ODM MPD-M. (C)

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In order to advise the President and the NSC as to the status of the Nation, the Director of the ODM will require from the National Damage Assessment System an overall picture of the scope of the damage on the population, the physical resources, and the military. The ODM will need an early assessment of the status of the work force and of the human resources available to reinforce both the work force and the military. Damage to the physical plant of industry, transportation, agriculture, and raw material resources and stockpiles will be required. It will also be necessary that the ODM be able to obtain assessments of what remains effective, and to what extent, of the overall national economy.

The total of the assessment information required by the ODM can then be analyzed and interpreted for the purpose of making an estimate of the Nation's remaining ability to fight the war to a successful conclusion provided there remains a capability to fight. As a result, the ODM should be able to develop from the damage-assessment information, a "big picture" estimate of the situation on the homefront. This picture together with the DOD's estimate of the military situation and the FCDA's estimate of the civil defense situation will provide the best available basis for the deliberations of the NSC and the decisions that will have to be made by the President.

FEDERAL CIVIL DEFENSE ADMINISTRATION

The FCDA is primarily interested in determining the results of the attack on the population and upon the physical structures within the target areas. In the case of population there are the questions relating to the dead, the injured, the availability of medical assistance, feeding problems, shelter problems, relocation, and so on. In the case of the physical damage, the question of cleanup, sanitation, water, fuel, sewage, and rehabilitation and related matters will require resolution. These and a host of related matters will require decisions based on accurate, timely knowledge of what actually happened. The success of the Nation's ability to survive and to recover will depend on the FCDA's ability to cope with these problems in such a manner as to maintain the morale of the people as well as a will to see the fight to a successful finish.

OTHER DEPARTMENTS AND AGENCIES

Each of the many other departments and agencies of the Government having operational responsibilities and functions as a result of the attack will require immediate results, expressed in terms suitable for their particular use.

WHOLE PICTURE

The net result of the requirements as previously listed for the assessment of the results of such an attack is a vast, comprehensive, and composite story of the overall damage to the Nation, plus an assessment of what remains alive of the population, the economy, and the military. To accomplish this the information produced by the National Damage Assessment System must range from a general overall picture suitable for consideration by the topmost echelons of the Government, all the way through a range of data which can provide what amounts to minute detail required by an office of some operating activity of an agency or department.

BOMB DAMAGE ASSESSMENT—THE KEY TOOL

Damage assessment, such as discussed in this paper, is thus seen as a key means to an end—the recovery and final victory. But, as of this writing, much remains to be done to make

the present systems evolve into an effective National Damage Assessment System capable of actually performing to its full potentialities. The indications are that, subject to the comments regarding the weaknesses, both technical and administrative, the damage-assessments systems being developed by the DOD, ODM, and the FCDA are more than likely to achieve a high potential to produce the results required.

CONCLUSIONS

It is concluded that with the attainment of a significant thermonuclear weapon and delivery capability by Russia, the results of a possible massive nuclear attack on this Nation, together with the resulting fallout, will be of such vast scope that a rapid, accurate means of determining the results of such an attack is a palpable need.

The magnitude of the damage and casualties defies assessment by any means now known except those provided by electronic computers.

Electronic computers can be programed to accept input data in two categories:

a. Prestrike data.—These, based on a wide range of kinds of data, can be placed on punched cards or magnetic tapes, provided only that such data be reduced to certain standard terms for which standards have been agreed upon.

b. Poststrike data.—While of relatively few kinds, these are far more difficult to determine rapidly and accurately so they can be made available for introduction into damage assessment systems.

A great mass of prestrike input data has in fact been reduced to standard terms and placed on punched cards and tapes for use in the damage assessment systems. The scope or range of kinds of such prestrike input data potentially covers virtually the entire scope of interests and responsibilities of all the various agencies and departments of the Government. It is continually being revised and added to, subject to availability of funds and personnel.

The provisions for providing poststrike input data constitute a definite technical weakness, or limitation, in the systems at the present time. Fallout pattern determination by electronic computation still has not yielded to solution. The NBS is working on this problem together with the Weather Bureau. Two projects are being actively prosecuted within the DOD to develop unattended monitoring systems which can provide accurate, rapid data regarding the location of GZ, the height of burst, and the yield of each weapon at each target.

The ODM will have a high-speed UNIVAC system installed in its dispersal location probably by mid-1957. The FCDA has its intermediate speed system in operation at its dispersal location. Work is currently being done to tie these systems together by electronic data transmission systems. The DOD has under study the question of establishing an electronic computer damage-assessment system. At present the prospects appear that it is quite likely that such a DOD system will be activated and that it will be located at a DOD dispersal location. Plans to tie the prospective DOD system into the ODM-FCDA data transmission system are under consideration.

There is no overall policy or guidance in the formal sense for the coordination and prosecution of development of damage-assessment systems. An ad hoc Steering Committee, however, currently coordinates the activities of the three sponsoring activities. The lack of formal policy and guidance does appear to cause some uncertainty as to what phases of the overall effort are the responsibility of each of the sponsoring departments or agencies. The chief "gray" area resulting from this situation is that of communications to provide for the introduction of poststrike data into the systems.

There are two (and likely to be three) electronic damage-assessment systems in being. This appears to be duplication. It is not so considered, however, since this dispersion of the systems represents "insurance" on one hand, while on the other, there are actually three rather well-defined areas of interest directly related to the interests and responsibilities of the department and agencies sponsoring the systems: DOD, ODM, and FCDA. Furthermore, the planned interconnection of these systems by means of data transmission systems permits the ultimate establishment of the National Damage Assessment System as planned.

The damage-assessment systems do, in fact, represent a key to the making of the needed decisions leading to postattack recovery and the further prosecution of the war following a massive nuclear attack upon this Nation. There is every prospect that, subject to the reduction or elimination of the present limitations and weaknesses in the systems, they can provide the information needed by the President and the NSC in their deliberations leading to crucial decisions. They can also provide a vast amount of data in varying degrees of detail which covers the range of responsibilities and interest of virtually all the departments and agencies of the Government.

RECOMMENDATION

It is recommended that the ODM undertake the responsibility for the overall formal policy determination and guidance so as to insure the orderly development and funding for the various systems which will eventually result in a truly integrated National Damage Assessment System.

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LOGISTICS RESEARCH PROGRAMS OF THE U. S. ARMY, THE U. S. AIR FORCE, AND THE U. S. NAVY*

Note by managing Editor: This article contains briefings which were presented in October 1957. It is published here to give readers some information concerning the logistics research programs of the U. S. Army, U. S. Air Force, and U. S. Navy.†

M. I. R.

TRANSCRIPT OF BRIEFING ON ARMY LOGISTICS RESEARCH

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About four years ago, the Deputy Chief of Staff for Logistics (DCSLOG) initiated a program of research on logistics problems by contracting with independent organizations for the conduct of such studies as appeared necessary. Contract studies were initiated for specific problems as the need arose, and generally, different organizations were selected for each contract. The result of this practice was separate contract efforts with several different organizations. The more important of these past contract efforts were as follows:

1. A basic study by George Washington University to isolate and define problem areas for further detailed study.

2. A sizeable effort, again by GW, to develop a Modern Army Supply System—popularly known as Project MASS.

3. Another was a study by Melpar, Inc. to develop a rapid method of computing materiel requirements and to develop a simplified supply control system.

4. Dunlap and Associates conducted a study to develop a rapid method of translating War Plans into Class IV and Service Troop requirements.

5. Dunlap and Associates also did the research on the ENVANAL project and developed a prototype system. This system indicates the effect on equipment operation under extreme environmental conditions.

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*Manuscript received December 1957.

†The following papers were presented, on 3 October 1957, to the coordinating Committee on Ordnance, Transport, and Supplies, Office of the Assistant Secretary of Defense (Research and Engineering).

6. Harbridge House developed the essentials for the Army Logistics Management Center which trains our key logisticians, both civilian and military.

7. Harbridge House also developed an Economic Inventory Policy which we are currently preparing to test.

These are representative of the kinds of research studies that have been initiated under the auspices of DCSLOG.

About a year ago we recognized that by following the practice of initiating contract studies as a problem demanded attention and by selecting, generally, a different organization for each study, our total effort lacked continuity, integration, and motivation. Accordingly, a study was made to develop a long-range program to eliminate these deficiencies. This study pointed up a need to make two basic changes in our logistics research program. First, a central coordinating agency should be established to insure integration of the individual efforts and motivation of the total effort, and second, the conduct of most contract studies should be concentrated in one organization to achieve continuity. The coordination function is now charged to one Branch in DCSLOG, but the manner in which we will achieve continuity through continued use of the same organization has not as yet been finalized. A number of methods have been considered, and the one considered by the DCSLOG staff to offer the most advantages was to implement a long-range program through a long-term contract with a single organization. This plan was not approved since the size of the effort did not justify such action. An obvious alternative is to incorporate the Logistics Research Program into the ORO work program. This method was considered in the original study but the separate contract method was felt to offer several advantages. However, since the size of the effort does not justify such a separate formalized program, the ORO method appears to be the most desirable course. The staff has arrived at this conclusion and is currently preparing to implement three studies by having these included in the ORO work program. The following four studies were recently approved for implementation and include those to be conducted by ORO:

1. First, a study of the problem of how to make the most effective use of the various Financial and other Management Control systems incorporated into the Army structure in recent years. This study has been initiated under contract with GW with the initial objective to define the problem and determine the method of attack.

2. Three studies to be conducted by ORO:
   First, Rebuild of Equipment
   Second, Plant and Production Equipment Reserves
   Third, Inventory Modernization

Four additional studies initiated in the past, and mentioned briefly earlier, must be considered as part of our current program.

1. One is the Dunlap and Associates study to develop a method and system to simplify and speed up the translation of War Plans into Class IV and Service Troop requirements. This contract expires at the end of Nov 57. The need for further research in this area has not as yet been determined.

2. A second is an ORO study, recently submitted in final draft, to develop "A System for Army Inventory Evaluation and Reporting." Initial staff evaluation of this study indicates it to offer considerable promise of providing answers to many of our difficult problems in the field of Inventory Modernization.

3. A third study by Harbridge House has developed an "Economic Inventory Policy." This is the problem of "Stockage Levels," "Safety Levels," "Reorder Policies," and the like. The policy proposed in this study looks very good and a contract has been awarded to Harbridge House for conducting a test of the policy.

4. The fourth study was by Dunlap and Associates to develop the ENVANAL system. As stated earlier, this system determines the effect on equipment operation under extreme environmental conditions. A prototype system has been developed and tested and looks very good. The Army staff in coordination with CONARC, is currently engaged in considering the desirability of developing the complete system for implementation.

Thus, our current program consists of some 8 studies. Our long-range program is still in the stage of formulation as pertains to specific study projects. In general, the long-range program consists of a continuing study by George Washington University to assist DCSLOG in analysing and defining problems for detailed study to include a determination of the method of attack. We plan to try this method of using a research group to assist us in gaining a clearer understanding of the problems confronting us in logistics, their relationships to each other, and how best to go about solving them. After all, once a problem has been clearly defined the most difficult phase is over and the battle is half won.

In addition to these contract studies many "in house" studies are constantly underway. Technical Services and CONARC as well as DCSLOG are constantly studying various logistics problems. This presents a rather broad-brush picture of Logistics Research in the Army as it is today.

BRIEFING ON THE LOGISTICS RESEARCH PROGRAM OF THE AIR FORCE

R. H. Brown
LT COL, USAF

The USAF program of logistics research involves both basic and applied research. Our objective is to discover and develop new principles or techniques which will allow the USAF system of support to be more responsive and flexible and still remain within budget limitations. There are definite payoffs to be had in the form of reduced stocks and improved support if we can more accurately calculate requirements, shorten our materiel pipeline time, and generally better our overall system of materiel management and inventory control.

Pursuit of this objective has led us into study areas concerned with the flow and processing of materiel information, the maintenance of inventory data, the distribution of peace and war stocks, the initial provisioning for support of weapons, the maintenance and repair of aircraft, and the physical movement of supplies throughout the world. We are not only studying these functional areas on an individual basis but examining their interactions and their relationship to each other. The interdependence of each functional area is such that no one area can be studied to the exclusion of all others. The overall aim of our research effort is to meticulously blend the new and improved features of each functional area to the extent that we will construct a modernized and well integrated system of support.

The RAND Corporation of Santa Monica, California, is currently our major contributor of research data in the logistics field. RAND is an independent, nonprofit organization whose

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principal business is long-range, scientific, military research, designed to aid Air Force decisions.

Although the RAND Corporation was formed in 1948, it was not until early 1954 that they were asked to look into the field of logistics. The Logistics Department of RAND is now engaged in the following major studies:

1. Analysis of Demand for Aircraft Spares: The understanding of demand is, of course, basic to any study of logistics and materially affects the design of an efficient system. From this research study which has analyzed the demand of aircraft spares for the B-47, B-50 and F-86 aircraft, certain conclusions have been reached relative to the uncertainty of demand and the correlation between the kinds, quantities, or dollar value of spares demanded, and the program activity of the aircraft. These findings are now being used in the development of improved methods of computing flyaway kits (mobile spares to support unit aircraft away from home base), levels of supply, and economic order quantities. Preliminary results in these areas indicate greater satisfaction of demands with less weight for flyaway kits, and fewer shortages with reduced order and handling costs for the stockage policy.

2. Initial Provisioning of Aircraft Spares: This study began approximately a year ago with an examination of how present Air Force provisioning was being accomplished. RAND analysis indicated that excess quantities of spares were being procured and that a large portion of the excess comes into the Air Force as part of the initial provisioning of a weapon system. As a solution to the problem, RAND's proposal is essentially to defer the initial provisioning decision, particularly with regard to Hi Valu items, as long as possible during which time more can be learned about spares demand, the flying hour program, and engineering changes.

3. Electronic Data Processing: RAND has been performing considerable research relative to the management of a materiel system which exploits the use of EDP. Project Electrologs which originated at RAND is a concept featuring world-wide management of any line item from one EDP point. An application of this concept to weapon system support of the B-52 and KC-135 is now underway at Oklahoma City Air Materiel Area. Also, RAND has a study nearing completion showing how data processing for many interrelated and complex materiel functions can best be integrated.

4. Modernization of USAF Airlift: The most recent work by RAND in the air transport field is an examination of dollar costs and manning requirements of a number of air transport fleets that might be procured and operated by the Air Force during a nine year period. The analysis recommends that procurement of newer type aircraft in certain instances would be more economical than maintaining the existing piston engine transport aircraft.

5. ICBM Logistics Support: Recently RAND organized a missile logistics study group for the purpose of analyzing logistics support for the ICBM. In August, RM-1920 was published. This particular study examines the support requirements and costs of varying operational requirements for the Atlas missile. In the Spring of '58, an experiment in the Logistics Systems Laboratory will be entirely devoted to the problem of ICBM support.

The Logistics System Laboratory was established at RAND in late 1956. The purpose and objectives of the Laboratory are set forth in Air Force Regulation No. 20-8. Briefly stated, the purpose is to document a laboratory simulation of the information processing and decision making activities of a modernized USAF logistic system. It is primarily intended to further develop ideas and concepts through the means of simulating the logistics operation. We believe this will help ensure greater reality and compatibility between research projects and the real world environment. The first official exercise of the Lab, called "LP-1," has just been

concluded. LP-1 lasted for 90 days and involved the full time of some 50 people, part of which were on TDY from AMC, ATC and ADC. This first formal experiment consisted of comparing a 1960 logistics system managed under proposed procurement, distribution, and data processing systems with a system managed under current practices. A movie depicting "LP-1" is now in production.

A research project we have budgeted for in FY 58 entails the development of an air cargo and ground handling system. It provides for the investigation and development of an optimum cargo handling system for the low and high floor level series of aircraft. The handling must be such that the loading, unloading, and securing of cargo can be accomplished within the time period required to service the aircraft. Plans are to expand this study into other areas to include the mechanization of terminals and finger piers and development of equipment to fill the gap between the aircraft and the terminal.

The Air Materiel Command has certain contracts for systems research which have been budgeted for in FY 1958. These contracts include one for streamlining dollar accounting procedures, one which deals with the development of depot maintenance standards, and another to help apply EDP to the Ballistic Missiles program.

This briefly covers the major research contracts we expect to fund in FY 58. The results of our research to date have been most worthwhile. As far as our plans for the future, we hope to continue the RAND effort in about the same order of magnitude for several years to come. Considerably more work is required in demand analysis and requirements determination. We are convinced that a refinement of our requirements and distribution policies will result in optimum support at minimum cost. Further research in the EDP area is needed if we are to take full advantage of advanced scientific techniques in the management of our resources. So far we have devoted only a small part of our research effort to the function of maintenance. This area is becoming increasingly important particularly with regard to the support of bases. We must explore the logistics peculiarities of missiles and relate them to the operational aspects of missile employment and the existing Air Force logistics structure. The Logistics Systems Laboratory is just getting underway, and we are convinced this will be a valuable research tool.

One of the significant areas which needs attention is air transportation. A study is required to develop new airlift planning factors for calculating resupply and deployment requirements for combat and support forces. The present airlift factors which were developed in 1954 need study and evaluation because of changes in concepts, support systems and new equipment.

The Air Force plans to continue logistics research in the areas just outlined as well as any other area required to provide a truly responsive and flexible system of logistics support.

BRIEFING ON THE LOGISTICS RESEARCH PROGRAM OF THE NAVY

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The logistics research program of the Navy has as its objective the application of the most modern concepts and techniques of business management and material support of the operating forces which can be brought to bear upon our problems. These new concepts and

techniques were not possible a few years ago, at which time all business management decisions had to be made on the basis of experience, judgment, and a great deal of intuition, assisted only by a few fairly crude yardsticks furnished by the accountants and statisticians to tell management how well it was doing after the fact.

Within recent years however, real breakthroughs in the fields of mathematical analysis, together with the development of the family of research disciplines known as operations research, have enabled us to bring to bear potent management tools for the purpose of making accurate predictions of requirements, calculation of inventory levels, design of efficient and economical distributions systems, and such related problems. However, these new techniques require such massive computation and calculation that their use would be of little practical value to operating organizations were it not for the concurrent development of electronic data processing machines and related rapid communications systems which enable the data to be collected and assimilated rapidly enough to furnish management with real guidance on a current basis.

These then—mathematical and operations research, electronic data processing machines, and rapid communication systems—are the three major tools of modern business management with which the Navy's logistics research program is concerned.

The Navy logistics research program, as is the case with all Navy research, is divided into two major elements: basic research and applied research. Basic research is the direct responsibility of the Office of Naval Research, which is also charged with the overall administration of the Navy's research program. Applied research in general is performed under the direction of the several bureaus of the Navy Department. In the logistics area, the Bureau of Supplies and Accounts performs the major portion of the applied research.

The Logistics Branch of the Office of Naval Research conducts the basic logistics research program. The word "basic" is a relative term. In the present context it embraces two levels, (a) the creation of theoretical foundation knowledge, i.e., "science for scientists," and (b) the development of new methods, techniques, and sophisticated procedures based upon sound theory and made available for applied research work. Since ONR's mission is to get research done for the Navy rather than to assume naval operating functions, the Logistics Branch does not evolve from its research undertakings any operating service functions. It does, however, assist operating agencies to exploit research results.

The Logistics Branch program can be divided into five main research areas: (1) Mathematics and Mathematical Statistics, (2) Econometrics, (3) Production Planning—Management Science, (4) Cargo Handling, and (5) Research on, or arising directly from, the Navy's own problems in logistics planning and control. These areas can be outlined as follows:

(1) Mathematics and Mathematical Statistics. Research here is concerned with the rigorous, somewhat abstract theories of decision processes—linear programming, general programming theory, game theory, and statistical decision theory. These theories are essential to many applications in transportation scheduling, warehousing, optimal resource allocation, and inventory management, to name only a few examples. For instance, at a conservative estimate, fifty percent (50%) of the existing theory behind the methods of linear programming has been developed in this ONR program. Much of this theory has been assembled for convenient access in the book "Linear Inequalities and Related Systems" edited by Professors Kuhn and Tucker as part of their contract work in this program.

(2) Econometrics. In this area where the scientific disciplines of mathematics and economics combine, the emphasis is again on decision-making theory, with particular refer ence

to conditions of uncertainty in input data and to the fact that decision makers are commonly not individuals, but organizations, often large ones. A good deal of attention is given to quantitative models of economic situations such as activity analysis, the Leontieff interindustry relations model, and the concept of capacity as it refers to industrial, or to military activities. One ONR contract in this area, supported by funds of the Department of Defense, itself, is exploring the practical applicability of linear programming and other quantitative techniques to critical materials stockpiling and alternate source decisions. Another contract produced the book "Theory of Inventory Management" by T. M. Whitin which has become a valuable reference in its field.

(3) Production Planning—Management Science. This third part of the ONR program concerns itself with the invention and development of mathematically based methods applicable to the planning, scheduling, programming, control, and balancing of production-type activities in relation to management policy, inventory, transportation and distribution factors, whether the operational context is military or civilian. Arrangements are made as practicable for research contract personnel to work with naval field activities having production-type functions in order to secure input of practical constraints which condition real problems and their workable solutions.

(4) Cargo Handling. In this area the research concerns itself with basic industrial engineering methodology as applicable to analysis of port operations, the land-to-sea or sea-to-land transfer of cargo with a view to reducing ship turn-around time and with particular attention to productivity measures, scheduling, packaging, and paper work. Time and motion and numerical simulation techniques have been worked out in connection with conceptual models of the processes involved and unique exploitation of electronic computation has been achieved.

(5) Navy's Own Problems. The Logistics Branch devotes about one half of its total budget to support of the Logistics Research Project at The George Washington University. The purpose of this contract is to devise and test methods for numerical simulation of naval actions in terms of logistics requirements, for measurement of the contributions of elements of logistics support to the success of naval operations, for determination of optimal stock levels for items whose future rates of use are uncertainly known, and for collection, reduction, and employment to naval advantage of data on the usage of materials by naval operating forces. This contract has the use of the Logistics Computer, an electronic device tailored in its design for use as a research tool in logistics and as a possible prototype logistics data processing system. Continuing low priority research on desirable machine characteristics results in progressive modification and improvement of this machine. The Logistics Research Project developed the Three Factor Method for logistics planning, a method including a systematic format with compiled and refined planning factors for its use, which makes explicit quantitative use of command judgment both in estimating requirements and in transmitting these estimates to other echelons. This Project cooperates with the Naval War College in the preparation for and play of the annual strategic war game, using the Logistics computer to simulate the logistics phases of a global war and to prepare planning materials for the assistance of the student officers. Progressively improved and more realistic models and methods are evolved from year to year; experience now comes to three years. The Logistics Research Project is also working in close cooperation with the Bureau of Supplies and Accounts program. Current cooperative research is devoted to the development and prospective operational testing of improved parts allowance lists for combat ships which will enhance operational endurance and efficiency.

The Logistics Branch program has been formulated to make the best possible use of severely limited resources in a wide open field. A superficial view produces the idea that logistics is military economics; it is really much more than that, embracing as it does a complete range from down-to-earth operations with hardware and hard-handed operators to highly subjective planning and forecasting functions. It is a newly opened field for research where it has only recently become possible to apply scientific methods. A foundation of theory and precise principles has had to be built from the raw material of science. This sort of thing represents the most efficient of the research processes and hence, the most inviting area for expenditure of research funds. However, logistics is also a field where research effort can still be naturally close to operations so that there is a good opportunity to reduce the normal time lag—which may often be up to fifty years—between basic research and applied methods. The Logistics Branch is attempting to facilitate the realization of this goal through its engineering and naval operations oriented tasks. Some outstanding successes have been achieved and others are in prospect. The work which is undone—not even attempted as yet, since the resources available are very slender in comparison with job—far exceeds that which is complete or underway.

As a specific instance of an effort which is urgently desirable but which it has not been possible to attempt, the Navy should have a laboratory in which techniques which are proposed, as a result of basic or applied research, can be tried out in safety but in reasonably realistic contexts. This would have the tremendous advantage of requiring such techniques to be spelled out in practical terms and of pointing up which of the always numerous operating obstacles are actually important and which are merely borrowed trouble.

The Navy's applied logistics research program is concerned with the development of a greatly improved, more effective, and less costly logistic support system. The basic elements of a long-range research program in this area were published in November 1956 in a document entitled "An Evaluation and Program for the Long Range Development of the Navy Supply System." This document has received the enthusiastic endorsement of most of the top leaders of the Navy.

The Navy Supply System presently has approximately $13.7 billions of material in store. It receives approximately $840 millions of material a quarter and expends some $880 millions a quarter at the present. There are 1,254,700 different standard stock items in the Navy Supply System.

These figures illustrate the tremendous business of the Navy Supply System. The management of this System, greater than the largest business organization in the United States, is confronted with many complex problems which are impossible of solution in the course of day-to-day operations.

Not only must the Supply System be up to date and advanced in materials research, but we must keep abreast of the advancements in the sciences of economics, mathematics, statistics, and electronic data processing which have resulted in the development of theory and techniques which have special application in the solution of complex problems peculiar to modern business management in all its ramifications. These problems have been generally defined and some effort has been made toward their solution. However, complete definition of these problems, their abstractions into soluble form, and the eventual development of readily applicable methods for dealing with them in a fairly routine manner is impossible unless maximum utilization of existing theory and techniques in the above sciences can be realized.

As an example the Navy is greatly concerned with the problem of maximizing the afloat endurance of the Fleet. One aspect of this is the provision of materials allowances which consist of proper quantities of the right items. This is a complex problem involving correlation analysis to disclose part to equipment and part to operational factors usage prediction ratios as well as distributive techniques. The payoff in this area, if our research is successful, will be much greater self-sufficiency of the Fleet at a much lowered inventory investment cost.

Another example is the development of new inventory management techniques. These involve tailoring item stock levels to reflect the variables of unit cost, relative military essentiality, manufacturing lead time, demand uncertainty, shelf-life, and obsolescence. They also involve balancing these factors against cost of procuring, storing, and distributing. Much has been done in this area, but very much remains to be done.

I could cite many other such problems requiring intensive applied research, but time does not allow. However, one I do want to mention is the whole host of problems related to the support of our new weapons and weapons systems. These problems, singly and collectively, greatly influence our ability to render material logistic support to the fleet within the limitations of the Nation's ability to provide the manpower and material that is represented by this logistic support. As each step toward the solution of these problems is attained, a significant result will be realized in more effective material logistic support and in reduction of the overall cost of supply operations.

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NONLINEAR NETWORK FLOWS AND CONVEX PROGRAMMING OVER INCIDENCE MATRICES*

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and

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INTRODUCTION

Problems involving network analysis arise in many industrial contexts. For example, interest often attaches to methods for characterizing the behavior of firewater safety systems in refinery operations. Flows and resulting pressures need to be ascertained under various patterns of input and withdrawal. Under certain conditions standard approaches such as Hardy Cross[1] methods and extensions of the usual Gauss-Seidel techniques, have been found difficult or impossible to apply in analyzing such systems. The nonlinear character of the branch head losses may cause lack of convergence of the solutions or require recourse to supplementary analyses of a complicated character.

Certain kinds of network problems have proved amenable to the usual formulations of linear programming.[2] Here it is proposed to utilize a somewhat different approach in order to provide a more general method of attack and to do so, moreover, in a way which provides access to the methods described in [4] for treating nonlinear problems as well. The essential ideas are as follows: (1) restatement of the original problem in terms of minimizing a separable convex[3] functional subject to a system of linear equations expressing the conservation of current entering and leaving nodes by branches;[4] (2) utilization of piecewise linear functionals to secure suitable approximations to the convex functionals; (3) using the "bounded variables techniques"[5] of linear programming in order to avoid expanding the number of vectors in an

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[1] See [1].

[2] See, e.g., W. Prager, [9] and the references cited therein.

[3] By "separable convex" function is meant one which is the sum of convex functions of one variable each.

[4] I.e., it is assumed that leakages, if any, may be ignored.

[5] Cf. [8].

"active" basis and also to avoid expanding the number of different coefficient vectors which require explicit expression. The resulting problem is thereby converted to an equivalent one which may be called the polygonal approximation to convex programming over the "incidence matrix" of a connected network. Special properties are then secured which simplify analysis and representation, and provide a basis for systematic elaboration.

INCIDENCE MATRIX

In engineering analyses of networks, it is customary to designate one of the two possible directions of flow in a branch as positive. The current is then allowed to have either positive or negative values. Positivity and negativity are then respectively associated with flow in the positive direction or its opposite.

Topological analysis utilizes instead the closely related notion of "incidence" of branches (or links) on nodes. Thus, the convention may be adopted, for example, that a branch entering a node, is positively incident on that node, a branch leaving a node is negatively incident on it, and a branch which does not meet a node is not incident on it. The incidences of branches on nodes may then be described by an incidence matrix consisting of numbers ϵ_{ij} representing the incidence of the j^{th} branch on the i^{th} node as follows:

$$(1) \qquad \epsilon_{ij} = \left\{ \begin{array}{l} 1 \\ -1 \\ 0 \end{array} \right\} \begin{array}{l} \text{if branch } j \text{ is positively incident on node } i. \\ " \quad " \quad j \ " \text{ negatively} \quad " \quad " \quad " \quad i. \\ " \quad " \quad j \ " \text{ not} \quad " \quad " \quad " \quad i. \end{array}$$

Since each branch is incident on precisely two nodes each column of the matrix will contain precisely two nonzero entries—a plus one and a minus one. If q_j designates the current in the j^{th} branch the net influx of current into the i^{th} node is evidently

$$(2) \qquad \sum_j \epsilon_{ij} q_j ,$$

where j ranges over all branches of the network. If the efflux out of the network at this node is E_i then this conservation of current at nodes (Kirchhoff's node law) may be written as

$$(3) \qquad \sum_j \epsilon_{ij} q_j = E_i , \quad i = 1, 2, \ldots$$

Thus, to express the conservation conditions, it is necessary only to form the incidence matrix column by column.

BASES OF COLUMN VECTORS FOR AN INCIDENCE MATRIX

As stated in (1) above, each column vector of such a matrix contains a plus one and a minus one and is uniquely associated with a branch of the network. The plus one corresponds to the node at the head of the branch and the minus one corresponds to the node at the tail of the branch. A basis of column vectors of the matrix is a collection of column vectors (or branches) such that, (1) every other column vector can be expressed as a linear combination of this collection, and (2) the number of vectors in this collection is the least with which this task can be accomplished. Clearly, a basis must include at least one vector incident (positively or negatively) on each node of the system. Otherwise, it would not be possible to express

(in terms of the basis) a vector associated with a branch incident on that node. To express a column vector (branch or node pair) in terms of the basis, it is necessary only to start with the head node of this branch and trace a path along branches associated with the basis to the tail node of this branch. The corresponding algebraic process consists of adding or subtracting the basis vectors associated with the branches thus traversed.

Consider, for example, the process of moving from one branch to another across a node. Both branches are incident on this node. If both have the same incidence, subtracting them will give a column vector with precisely one "plus one" and one "minus one" associated with the two end (or exterior) nodes of these two branches; if the branches are oppositely incident on a node, an addition will be performed to attain a vector corresponding to the end nodes. Thus, traversal of the path from node to node is associated with addition and subtraction of vectors so that as any particular node in the traversal is reached the cumulant vector has nonzero entries (of opposite sign and unit magnitude) corresponding precisely to the end nodes of the completed portion of the path.

The collection of branches corresponding to the basis can contain no closed path (or cycle). Otherwise it would be possible to express one of the branches in the cycle in terms of the others by the process just described. Every expression containing the vector of this branch could then be replaced by an expression not containing it. The supposed basis would then contain more than the minimum number of vectors required and would therefore not be a basis.

A connected system of branches and nodes containing no cycle is, in topology, called a "tree." Thus, a basis for the incidence matrix of a connected network consists of the column vectors associated with a tree which contains every node of the network. An easy method is thereby provided both for securing a basis and for expressing all vectors in terms of it.

DETERMINING THE "EVALUATORS"

The "evaluators"—the ω vector[6]—may be determined in a manner similar to the way in which the "row-column numbers" are determined in distribution type models of linear programming.[7] In the model which is being developed ω_i is associated with the i^{th} equation. The value of any one such ω_i may be selected arbitrarily because the system of equations (3) has the same degree of linear dependence as the standard distribution model.[8] The other ω_k are then quickly determined since each basis branch consists of a node pair and the calculations are initiated at node 1 and proceed away from it to all the other nodes of the network along the appropriate basis branches (hence column vectors).

EXTREMAL PRINCIPLE AND THE FUNCTIONAL

In the case of flow problems to be considered here, it is assumed that to each branch j there corresponds a resistance function $r_j(q_j)$ such that

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[6]Any solution of $\omega^T B = c^T$, where B is the basis at the current iteration. See [7], for example.

[7]In the following discussion some familiarity with linear programming is assumed. See, e.g., [3] and [5] for elaboration.

[8]This may be seen from the fact that the number of branches in a basis tree is one less than the number of nodes. In physical terms this corresponds to the fact that the voltage differences rather than absolute voltages are determined by the currents in a network.

$$R_j(q_j) \equiv \int_0^{q_j} r_j(q)\, dq \tag{4}$$

is a convex function of q_j. For example, in the firewater system problem

$$r_j(q) = \alpha_j \operatorname{sgn}(q) \times \left|q\right|^k, \quad k > 1$$

and

$$R_j(q) = \frac{\alpha_j}{k+1} \left|q\right|^{k+1} \tag{5}$$

The convex programming problem, or extremal principle, characterizing the network flow is

$$\text{min.} \sum_j R_j(q_j)$$

subject to (6)

$$\sum_j \epsilon_{ij}\, q_j = E_i, \quad i = 1, 2, \ldots$$

It may be established[9] that any optimal solution to this problem automatically satisfies the "head-loss," or Kirchhoff cycle conditions—viz, the algebraic sum of the head-losses around any closed loop consisting entirely of current carrying branches is zero. This principle thus achieves an essential simplification of the problem because in arriving at a solution it is only necessary to consider individual branches with the highly nonlinear loop conditions being transferred to the functional[10] in a much less recondite form.

APPROXIMATION OF THE FUNCTIONAL

To any desired degree of approximation each $R_j(q)$ may be replaced by a piecewise linear function so that the total, $\sum_j R_j(q_j)$, may also be approximated.[11] The resulting problem may then be replaced by an equivalent linear programming problem using the bounded variables techniques in order (a) to avoid proliferation of the number of vectors in the active basis and (b) to avoid increase in the number of structurally distinct vectors.[12]

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[9]The authors wish to thank the referee for stimulating a sufficiently general proof of this (which will be published elsewhere) based on extending the Kuhn-Tucker theorem for this situation. He pointed out that Duffin, Birkhoff, and Diaz required $r_j(q)$ continuous and strictly increasing in their proofs.

[10]It should be noted that the objective here is to obtain a simulation and that this is accomplished by optimization with resulting gains in efficiency for the desired simulation. For mathematical proofs of this principle under heavier restrictions than are necessary, see [10] and [11].

[11]Additional research which eliminates the need for this approximation step will be reported in a subsequent paper. The method to be reported consists essentially of implicit refinement to the limit of the approximation process and of short-hand rules so that no approximation needs to be dealt with explicitly. This, of course, replaces the linear iterations with a nonlinear counterpart.

[12]See [4] for a general description of this method and others for approximating a nonlinear functional by a piecewise linear one.

In detail, let

$$q_j = q_j^+ - q_j^- ; \; q_j^+ , \; q_j \geq 0$$

and further let

(7)
$$q_j^+ = \sum_k \Delta_{jk}^+ , \; q_j^- = \sum_k \Delta_{jk}^-$$

$$0 \leq \Delta_{jk}^+ \leq \Delta_k^+$$

$$0 \leq \Delta_{jk}^- \leq \Delta_k^- .$$

Then, the approximating problem to (6) assumes the form[12]

$$\text{min.} \sum_{j,k} \left[\rho_{jk}^+ \Delta_{jk}^+ + \rho_{jk}^- \Delta_{jk}^- \right]$$

subject to

(8)
$$\sum_j \epsilon_{ij} \left[\sum_k \left(\Delta_{jk}^+ - \Delta_{jk}^- \right) \right] = E_i$$

$$0 \leq \Delta_{jk}^+ \leq \Delta_k^+$$

$$0 \leq \Delta_{jk}^- \leq \Delta_k^-$$

where i, j, and k are each assigned their respective ranges.

METHODS OF COMPUTATION AND ILLUSTRATION

The method of computation on the network graph may be summarized as follows (the analogous steps on a tableau will be clear from the illustration). One starts with a solution of the node conditions admitting only branches which form a basis. The ω-vector is then calculated using the active basis, and from it, the Z_j is simply $\omega_h - \omega_t$, where node h is the "head" of branch j and node t is its "tail" as determined by its orientation. These are compared as usual with the c_j (in equation (8) the ρ_{jk}) to determine optimality or nonoptimality. If nonoptimal the vector to "come in" to the basis is expressed in terms of the active basis simply by going from the tail to the head of the come in branch on active basis branches. Those branches whose orientation is opposite to this direction of travel will have their "current" increased in the new solution, those with same orientation will have it decreased. In other words, the special structure of this problem is used to generate the information necessary to a simplex (or dual) procedure at each stage rather than using the general algorithm and storing additional information.

The example of Figure 1 may be used to provide a simple illustration for the computational routine to be employed. In this Figure an influx of 10 units is being put into the system at node 1 and effluxes of 2, 5, and 3 units, respectively, are being drawn from nodes 5, 7, and 9.

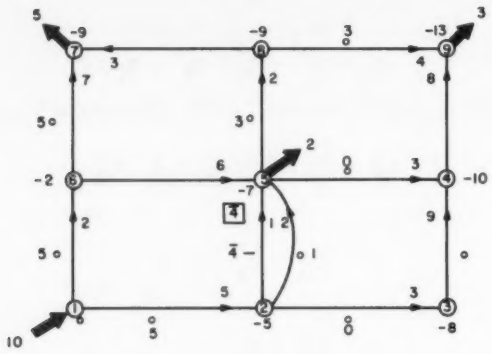

Figure 1 - Network Graph

The coefficients (here constants) for the variables which enter into the functional are shown inside the diagram opposite each arrowhead.[13] A physical analogy for these coefficients is found in the resistances, or head losses, over these branches in the circuit which are here assumed to be constants. The resistances are also assumed to be the same irrespective of the direction of flow.[14] Finally the branch 2, 5 is assumed to be characterized by a resistance of one up to 4 units of current and a resistance of two for greater currents. This can be visualized most easily by inserting two branches between nodes 2 and 5, designating the first as $\overline{2, 5}$, to indicate that it is limited in the amount of flow it will admit and the second by 2, 5 to indicate the absence of such an upper bound on the possible values of the flows through this (the latter) branch. The objective then is to characterize the current flows by minimizing the value of the functional overall branches of the network, taking account of the fact that $\overline{2, 5}$ is limited to a capacity of four units, as indicated by the value $\boxed{4}$ alongside its arrowhead, and that further flows along the branch will encounter the increased resistance associated with 2, 5.

Table 1 contains the corresponding incidence matrix with nodes entered in the stub and branches at the head. Each column is composed of the values ϵ_{ij} as described in section 2, above. The external influxes and effluxes for the system are noted at the right of the matrix in the column labelled "stipulations." Each row of the matrix then represents an equation which is a Kirchhoff node condition to describe the conservation of current. At the top of each branch is a value, c_j, to be entered into the functional and drawn from the amounts entered at the arrowheads on the branches of Figure 1.

Computation may proceed by means of either the Table or the Figure. For purposes of exposition, both devices will be used.

First an active basis is selected. To guarantee that a basis is, in fact, selected it is necessary, as previously described, only to choose branches in such a manner that all nodes in the system are attained without loops. Hence computations may commence by selecting a

[13] In the actual computations, it is advisable to utilize different colors to distinguish between these values, the programmed flows and the values for the ω vector.

[14] It would be possible to consider these resistances as different, depending on the direction of flow. Uni-directional flow could then be assured by blocking out counter flows with heavy resistance penalties.

TABLE 1
Incidence Matrix and Computing Routine

TABLE 1
Incidence Matrix and Computing Routine

ω Components (basis set)	Node	Functional coefficients / Branch	5	2	3	1	2	9	3	8	6	2	7	3	4	Stipulations
			1.2	1.6	2.3	$\overline{2.5}$	2.5	3.4	5.4	4.9	6.5	5.8	6.7	8.7	8.9	
0	①		1	1												= 10
-5	②		-1		1	1	1									= 0
-(5+3)	③				-1			1								= 0
-10 ~~-(5+3+9)~~	④							-1	-1	1						= 0
-7	⑤					-1	-1		1		-1	1				= -2
-2	⑥			-1							1		1			= 0
-9	⑦												-1	-1		= -5
-9	⑧											-1		1	1	= 0
-13	⑨									-1					-1	= -3
Basic indicators			°	°	°	$\overline{\ }$	°	°				°	°		°	
Program			5	5	0	4	1	0				3	5		3	
z values, nonbasis vectors						2			10	-4	5			0		
Basic indicators			°	°	°	$\overline{\ }$	°		°			°	°		°	
Program			5	5	0	4	1		0			3	5		3	
z values, nonbasis vectors						2		2		3	5			0		

promising basis set for the purpose of minimization. If this set is not optimal, a series of systematic iterations will secure such a set in a finite number of stages.

The basis thus initially chosen is indicated by the dots centered on the outside of the selected branches. It consists of the following branches: 1, 2; 1, 6; 2, 3; $\overline{2, 5}$; 3, 4; 5, 8; 6, 7; and 8, 9; as shown in Figure 1.[15] This set of dots is repeated directly below Table 1 in the first row labelled "Basis indicators." It designates the collection of column vectors which constitute an active basis. Immediately below these dots the row labelled "Program" shows the (unique) current flows which are readily ascertained from Figure 1 by reference to the branches in the basis and the influxes and effluxes.

Note that $\overline{2, 5}$ and 2, 5 are linearly dependent. Hence, at most, one of these vectors may enter into a basis. The other elements of a basis having been chosen, the total amount of current to flow between nodes 2 and 5 is uniquely determined. Since the objective is minimization, the optimal choice consists of filling up each such set of branches in the order of successively increasing c_j's. Thus, the flow of 5 between nodes 2 and 5 is allocated as follows: First as much as possible of this flow is allocated to $\overline{2, 5}$. Whenever such a link is used to capacity with overflow to one of its parallel links, such as 2, 5, it is not counted as part of an active basis.[16] For this reason $\overline{2, 5}$ receives a bar rather than a dot in the row of Table 1 labelled "Basis indicators," as well as alongside the program value $\overline{4}$ in Figure 1. The bar (replacing the dot) indicates that this branch is at capacity (with overflow) and is, therefore, not in the basis.

By virtue of the linear dependence of the Kirchhoff node conditions, exactly eight vectors here constitute a basis. The above method of choosing between $\overline{2, 5}$ and 2, 5 resolves the question of choice among links joining the same node pair, only one of which can be in an active basis.

To determine whether the programmed amounts are optimal, the methods of linear programming are now applied. Values, ω_i, are determined, one for each node of the network, as follows.[17] One such value may be chosen arbitrarily. Here the first such value, zero, is selected at node 1, and entered to the left of this node (under the ω components) in Table 1.[18] The remaining ω components are now uniquely determined by the condition that each pair of values ω_i and ω_k when multiplied by the corresponding elements of each basis vector and summed, must equal the value, c_j, located at the top of each such vector. Call the sum z_j which results from algebraically adding each pair of ω's so that the condition to be satisfied is $z_j = c_j$. Noting that zero appears as ω_1 opposite node 1 and that $c_{12} = 5$ it is apparent that $\omega_2 = -5$. Similarly, since $c_{16} = 2$ it follows that $\omega_6 = -2$ in order to obtain $\omega_1 - \omega_6 = 1 \times 0 + (-1)(-2) = z_{16} = c_{16} = 2$. Next, the value $\omega_3 = -8$ is determined from $\omega_2 - \omega_3 = z_{23} = c_{23} = 3$. Since $\omega_2 = -5$ it is evident that $\omega_3 = -(5+3)$, as shown, to achieve the desired equality.[19]

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[15] The dot on branch 5, 4 should be ignored at this stage. It enters at a subsequent iteration when 3, 4 is eliminated, as will be explained subsequently.

[16] See [3] or [7].

[17] It may be helpful to regard these values as tentative node potentials in terms of which the flows are determined.

[18] Note that these values may also be determined directly from Figure 1. Thus, a value of zero is inserted at node 1. When c_{12} is subtracted from this amount, the value of ω_2 at node 2 is secured. Similarly, the value -8 for node 3 is obtained by subtracting c_{23} from ω_2, and so on.

[19] Note that these values may also be determined directly from Figure 1. Thus, a value of zero is inserted at node 1. When $c_{12} = 5$ is subtracted from this amount the value $\omega_2 = -5$ at node 2 is secured. Similarly the value $\omega_3 = -8$ for node 3 is obtained by subtracting c_{23} from ω_2, and so on.

Continuing in this fashion utilizing only vectors designated by a dot (and therefore in the basis) the remaining values in the left hand stub of Table 1 are thereby determined.[20]

All vectors in the active basis, therefore, necessarily have $z_j - c_j = 0$. If the program is optimal those vectors which are not in the active basis have values $|z_j| - c_j \leq 0$[21] and bounded variables, such as the value for 2, 5, will have values $z_j - c_j \geq 0$.

If any value is located which does not meet these conditions, it is necessary to iterate further. In the present instance $|z_{54}| > c_{54}$. This vector is therefore brought into the basis, as indicated by the upward pointing arrow just below the first row of "z values" of Table 1. To determine which vector is to be eliminated, recourse may be had to Figure 1. Since 5, 4 is being brought into the basis, the node at 4 can be secured via this route. That is, the vector 5, 4 may be stated in terms of the basis by tracing the unique path along the basis tree either from head to tail or tail to head. Thus, 5, 4 utilizes 5, 2; 2, 3; and 3, 4. The first is accorded a minus sign as the negative of 2, 5 and the latter are accorded positive signs. Only the latter (the ones accorded positive signs) are considered for purposes of designating the vector to be eliminated from the basis when 5, 4 is inserted. By recourse to standard procedures, including known methods of resolving degeneracy,[22] the vector 3, 4 is eliminated from the basis and 5, 4 inserted instead.

Crossing out the programmed value of zero along 3, 4 and inserting it instead at 5, 4, all other flows are left undisturbed at their previous values. Also, the only value of ω_i which requires alteration, is the one at node 4. Starting with $\omega_5 = -7$ and subtracting $c_{23} = 3$ from this amount, the new value $\omega_4 = -10$ is secured to replace the old one of -17, as shown in Figure 1.

The second row of dots in Table 1 indicates the new basis with the programmed amounts entered directly below the dots. To check for optimality, all vectors not in the active basis are evaluated as before. The test for optimality being successfully passed, as indicated by the numbers in the z value row at the bottom of the Table,[23] the problem is solved with the current flows as indicated in the "program" row.

The methods for convex programming over incidence matrices here developed, may be generalized to what may be called matrices of incidence type with nonzero entries other than ± 1. Specifically they may be considered as matrices for node conditions which generalize the Kirchhoff node conditions in that the current in a particular branch is to be multiplied by a number associated with the branch before entry into the node. Such extensions are important, for instance, in problems of accounting and financial analysis, and are encountered even in very simple cases such as the ones described in [6], the further refinement of these methods[24] to the exact functions and involving nonlinear iteration steps,[25] is also important for dealing with problems of risk and uncertainty. These topics will be treated in subsequent papers.[25]

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[20] For a discussion of the theory underlying these procedures, see [3].

[21] Since flows may be in either direction, it is necessary either to use the absolute value of z_j or else to take account of the direction of flow. In links such as $\overline{2, 5}$, the direction of flow must be considered. The z_j which results from a negative flow, is simply the negative z_j for positive flow. Thus, it is unnecessary to carry two columns. In either case—positive or negative flow—on a capacitated link the true z_j must be greater than or equal to the corresponding c_j.

[22] See [2].

[23] I.e., $|z_j| \leq c_j$ for all nonbasis vectors and $z_j \geq c_j$ for $\overline{2, 5}$.

[24] See footnote 11, page 234, supra.

[25] E.g., A. Charnes and C. E. Lemke, "Continuous Limit Methods in Mathematical Programming," ONR Research Memo. No. 1, Systems Research Group, Northwestern University (forthcoming).

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DESIGN CHANGE IMPACTS ON AIRFRAME PARTS INVENTORIES*

James W. Petersen† and Wilbur A. Steger

INTRODUCTION

Various theoretical systems have been developed which describe the requirements of an "optimal" inventory policy. Such systems normally require as inputs costs of reordering, stock depletion costs, and expenses associated with holding an item in inventory. Unfortunately, many of these cost elements are not readily available in the present military logistics system; as a result, it has not been possible to test these optimal inventory systems, or for that matter, to evaluate present military inventory policy. This study is an attempt to provide part of the information needed to assess the costs of holding a spares inventory, namely, the cost of airframe inventory which becomes obsolete as a result of design changes. Such information, for example, would be extremely useful in determining a suitable spares procurement policy for a new military weapon. The central problem of initially providing spares for this new weapon is that, at the time the first spares' buying decision is made, spares requirements are so uncertain. The phase-in of a typical Air Force weapon system occurs over a two-to-four year period. Airframe manufacturers, however, insist on production lead times of from one-to-three years on major components (which are the expensive ones). Consequently, the Air Force has, until now, been required to make its buying decisions several years prior to the delivery of the first end item, and several more years prior to the point where any sizeable block of experience with the end item has been obtained. It is not just that wearout rates on parts are uncertain. At the time the buying decision is made, the final configuration of the weapon is only a rough guess. About the only thing known with certainty is that parts purchased will later require modification or will become obsolete as a result of design change. The Air Force spares' provisioner is asked to state the requirements for a part with an unknown failure rate and for a somewhat uncertain model configuration.

As a solution to this problem it would, of course, be desirable to do anything that might reduce requirements uncertainty. There are some things that might be done in this regard, e.g., demand experience on similar parts used on similar aircraft could be used. In general, however, it would be a mistake to overemphasize the gain to be realized by working on improved forecasting methods alone. The technological race will be with us for some time to come—we will be provisioning new and changing types of weapons. In short, as long as procurement procedures and policies require that the principal procurement decision be made

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years before the requirement exists, the initial buying decision will continue to be dominated by uncertainty.

The kind of system needed to minimize the adverse effects of poor requirements predictability is fairly clear, at least in concept. To take a limiting case, if the production process were perfectly flexible and highly responsive, the Air Force would order a part whenever a requirement arose. In this way demand uncertainties could be shifted backward and absorbed by the production process. Since such a perfectly responsive production system does not exist, and since weapons grounded for lack of support are of no use, it is necessary that some minimum support be ordered. Over and above this minimum, however, the object of the game should be to "live hand to mouth;" a policy would be pursued of placing numerous small stock orders to cover uncertainty in the immediate future and thus have a chance to let unfolding-demand experience be reflected in follow-on orders. By not placing a large initial order, it would be possible to shift part of the requirements' uncertainty back to the contractor's production line.

Naturally, such a system change would not be costless. Only at some additional cost could the contractor's ability to turn out some parts on short order be increased. In other words, while it may be difficult to cut lead times for all items, it may be possible to evolve a system which could provide selective items on a responsive basis. A differential provisioning policy might be envisioned in which low cost items were initially procured in large amounts—possibly even "life-of-type"—so as to minimize reordering and handling costs, while efforts would be made to obtain shorter procurement lead times on high-cost items.

The value of such a system revision depends, among other things, upon the savings that would result from a decreased impact of design change obsolescence upon spare parts inventories. Since high cost spare parts, while a relatively few items, account for the bulk of the inventory dollars, the saving potential (from this source alone[1]) would be high. To date no estimate of this saving, which might be called the "value of procurement deferral" has been made. Since the absence of an estimate of spares' obsolescence risk is partially responsible for this important gap in logistics' knowledge, the purpose of this study is to provide such an estimate.

SCOPE AND METHOD OF STUDY

The risk of design change varies between different classes of military property; therefore, the results of this study which relate solely to airframe property (aircraft structural components) should not be generalized to other dissimilar types of equipment. This report is not a general study of the costs of technological change. It considers only costs of the engineering obsolescence risk which a military service takes when it purchases quantities of airframe spare parts. Two other types of costs of technological change not covered in this report include: (1) The administrative costs of introducing and keeping up with design changes. (The constant stream of engineering changes for any weapon multiplies the complexities of managing the logistics system); and (2) The costs of aircraft grounded while modification is being accomplished.

The obsolescence measures developed are based primarily upon the study of one Air Force weapon, the F-94 series of aircraft. This aircraft was chosen for several reasons, not

[1]Other types of savings, such as a decrease in interest charges and warehousing costs, also are indicated for such a system of "deferring the procurement" of expensive spare parts. A future RAND report by the authors explores this area more fully.

the least of which was the fact that available data were more complete for this model than for others. In addition, the F-94 is currently being phased out of first line operational use,[2] during which almost all of the engineering changes affecting a given series aircraft occur. Data were also collected on the F-100 and the B-52 model aircraft; but, since these aircraft types are currently being phased into the Air Force inventory, the results are limited and serve primarily as a check on the estimates obtained for the F-94.

The study is mainly concerned with weapon modifications introduced by the airframe contractor; design changes affecting Government Furnished Aeronautic Equipment subsystems, such as engines and many of the electronic components, were excluded. Naturally the airframe modifications have an impact on other Air Force property classes, but the dollar value of such impacts is small and centered in such low cost classes as aircraft "hardware." No effort was made to segregate these costs.

The stock of spare parts effected by design changes includes all stock already delivered to the Air Force, plus all parts so far along in the production process that corrective action either is needed to bring the item to the latest configuration or is no longer possible. The impact of specific design changes can take the following lines:

1. Engineering obsolescence costs, defined as the dollar value of spares made totally obsolete by a design change.

2. Modification costs, which include both the material and labor required to rework a spare part to a new configuration.

3. Contractor "design-termination costs," which include the cost of spares material or work in process that must be discarded or reworked as a result of a design change.

In addition to the costs listed above, there is another cost which arises when the applicability of a spare part becomes limited as a result of a design change. For example, a part may be "common" to several different models or series of aircraft; but a design change on a particular series of aircraft will curtail the usefulness of the part. The item is not made obsolete, since it can still be used on other models; but the Air Force has on hand more of the item than is necessary to support the aircraft for which the part is still functional. This cost resulting from a reduction in applicability will eventually show up in the form of "terminal obsolescence," where the end item for which the spare was originally purchased phases-out of the Air Force inventory before the spare is used.

A true measure of the impact of design changes would include an estimate of the cost of limited applicability; but this cost is difficult to measure. In order not to neglect limited applicability altogether, two estimates of spares cost of design change were made. First, the direct spares cost of the change was measured, that is, engineering obsolescence, modification costs, and contractor design-termination costs. This estimate tends to understate the cost of design change, since it does not include the cost of limited applicability. In the second estimate, design change cost includes the full value of all delivered spares whose applicability is in any way limited by a change in design. Obviously this method overstates[3] the spares cost of design change, since many of these parts might still be useful on some other series of aircraft. Thus, in concept, method (1) puts a lower limit upon the cost of change—method (2), an upper limit. The true cost probably lies between these two estimates.

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[2]First line aircraft are those capable of being used to perform critical and essential Air Force missions.

[3]The "overstatement" is only a conceptual one, since some data gaps are known to exist in both estimates. The effects of these gaps will be taken into account, below.

The specific data used in this study are examined, in some detail, below. However, the methodology used can, doubtlessly, be applied to other areas. In general, the data requirements of the components of the problem are as follows:

1. Engineering obsolescence costs. The dollar value, preferably at current market prices less scrap value, of all spare parts' inventories that are condemned (i.e., the spare part is no longer applicable to any end item) as a result of the design change.

2. Modification costs. The dollar value of "retrofit kits" purchased by the military service for the purpose of reworking the spare parts' inventories that were not condemned by the design change, plus the dollar cost to the military service of all labor required to perform this "retrofit" operation on the spare parts.

3. Design termination costs. The cost to the military service resulting from the effect of a design change upon the raw material and purchased parts (in the contractor's plant) which were about to be made into spares at the time of the design change, whether the design change causes a condemnation of this material or merely necessitates its rework.

Before turning to the specific results of the analysis, it is desirable that the reader obtains some idea of the numbers and types of design changes which affect a military aircraft.

TYPES AND TIMING OF DESIGN CHANGES

Various kinds of engineering changes affect modern aircraft. While only a relatively few of these modifications affect the inventories of delivered spare parts, almost all of them are important from a logistics viewpoint.

Figure 1 shows the number of design changes processed monthly over the period January 1955 to March 1956, by the Lockheed Engineering Department for five Lockheed aircraft, the F-104, T-33, T-2V and P-2V (Navy), and the 1049 (an older version of the commercial Constellation).[4] The T-33, P-2V, and 1049 aircraft represent older models, their basic

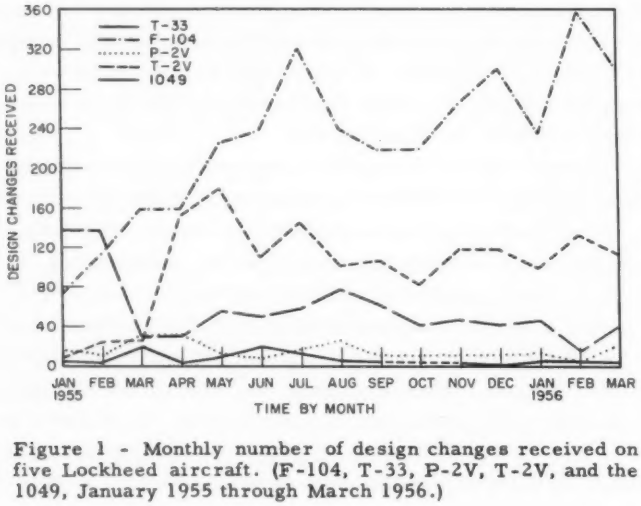

Figure 1 - Monthly number of design changes received on five Lockheed aircraft. (F-104, T-33, P-2V, T-2V, and the 1049, January 1955 through March 1956.)

[4]Unfortunately, similar data for the F-94 aircraft were not available. However, discussions with Lockheed personnel indicate that the characteristics of the types and timing of design changes are very similar from aircraft to aircraft.

design having been determined a decade or more in the past, (the T-33 is a development of the F-80). While the T-2V is a T-33 altered for Navy carrier operations, major modernization and modification have gone into its design; therefore, the F-104 and, to a lesser degree, the T-2V can be classified as newly designed aircraft.

Figure 1 shows that the number of engineering changes is greatest for the newer models of aircraft.[5] More than three-fourths of all the changes processed during this time period occurred on the F-104 and T-2V, whose first production models were completed during December 1955. This suggests that the number of changes for any one aircraft model is greater during the early stages of its production. It is interesting to note, however, that problems requiring some modification in the design of even the older aircraft continued to arise.

The design changes plotted in Figure 1 fall into two general types. First are those which can be performed within the aircraft's original contract specifications and with limited money expenditure. The aircraft contractor is given a rather free hand in introducing these modifications, and they make up the bulk of the changes listed in Figure 1. As far as possible, the contractor attempts to make these modifications with a minimum disruption of the production line. In many cases the aircraft will continue to roll off the production line without incorporating the new improvement, while the contractor uses up the material and parts ordered or produced to meet specifications of the older configuration. From a logistics point of view, such design changes should not greatly influence spare parts inventory, for with the policy of concurrent delivery of spares,[6] the stock in the hands of the Air Force will be applicable to the aircraft delivered to date. The aircraft incorporating the new design will enter the system with the spares support associated with the new design.

The primary impact of this type of design change is in the added administrative problems that limited applicability brings about in cataloguing and record keeping. Furthermore, as the sheer numbers of spares line items increase, the problems of requirements determination become more complex. However, the complexity is primarily a matter of increasing the numbers of requirement computations rather than the difficulty of each computation. Naturally, this holds only if the data processing system underlying the requirements determination is capable of correlating each spare with all the end items to which it is applicable.

More serious design changes are those which either alter the basic specifications of the aircraft or entail appreciable costs. In either case, the contractor does not wish to go ahead with the change without specific approval from the Air Force, since major modification to the aircraft and the spares already delivered may be involved. Most of the design changes that seriously affect delivered spares fall into this category.

Reasons for airframe modification vary: (1) the prime contractor may submit a change which he feels will improve performance or simplify production; (2) contractor vendors may alter their components so that airframe modification becomes necessary; (3) customer experience or change in requirements may indicate the desirability of a change. Figure 2 indicates the percent of the number of engineering changes by cause for the T-33 and T-2V for the period January 1955 to March 1956. Of interest is the fact that the relative importance of contractor

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[5]While total costs of these design changes were not available, Lockheed personnel in charge of design change were convinced that Figure 1 would continue to reflect the same picture even if costs of design change, rather than their number, were used. In fact, their feeling was that the cost per design change was higher, during the period, for the F-104 and T-2V.

[6]With concurrent spares' delivery, the spares' order is delivered in the same proportion as the end item they support; for example, when half the aircraft are delivered, half of the spares initially purchased for these aircraft is also delivered.

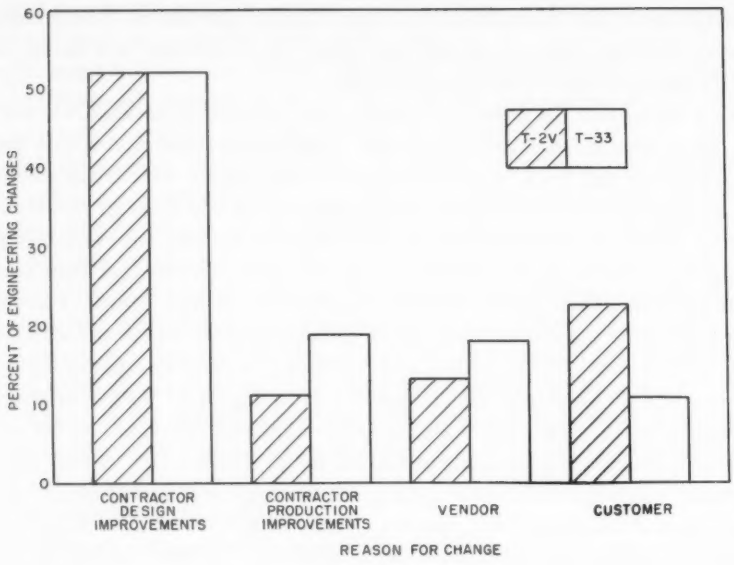

Figure 2 - Percent of engineering changes by reason for change. T-2V and T-33 aircraft, January 1955--March 1956.

design improvements is similar between these two models, even though the T-2V is a much newer model having a greater number of design changes.[7] The T-2V shows a larger number of customer, rather than vendor or production improvement, changes. No data exist showing whether this difference arises because of the age difference of these two aircraft models.

DESIGN CHANGE IMPACTS ON DELIVERED F-94 AIRFRAME SPARE INVENTORY

The F-94 series of aircraft was purchased under five separate contracts negotiated between 1949 and 1951. Approximately 800 F-94 aircraft were delivered between early 1950 through mid-1954. The airframe spares initially purchased for these aircraft amounted to approximately $64.3 million.[8]

To analyze design change impacts on F-94 spares the following components were estimated:

a. Engineering obsolescence
b. Spares modification
c. Contractor design-termination costs
d. Limited applicability

The results obtained and the method used to measure each of these types of spares design change impact are discussed in turn.

[7]Lockheed officials believe that the relative importance of the various causes of design change is very similar from model to model.

[8]Obtained from AMC Budget Program Summaries for the appropriate years.

Engineering Obsolescence Costs

The basic data used to evaluate the dollar value of airframe spares made totally obsolete as a result of design change were the 151 Time Compliance Technical Orders[9] issued by the Air Force for the purpose of introducing changes on F-94 aircraft. Design Change Technical Orders, known as T.O.'s or T.O.C.'s, are usually prepared by the manufacturer after negotiations with the Air Force. Each T.O. will describe the change to be accomplished, the serial numbers of the aircraft affected, and the dates within which the change should be made. Normally a design improvement will specify a 24-month span within which to accomplish the change. A safety change, on the other hand, specifies immediate compliance.

The T.O. will specify whether or not the design change affects the end item in such a way that the usability of delivered spare parts is also affected. In each technical order, there is a separate list of spares that are affected as well as the disposition of such parts. Disposition possibilities fall into one of the following categories:

a. Condemn and dispose as surplus
b. Retain for use on unmodified aircraft
c. Retain for use until modification of the aircraft is accomplished
d. Modify per instructions contained in the technical order

Spare parts made obsolete through design change were defined as the sum of categories (a) and (c) above. The dollar value of these condemned parts was obtained by multiplying the on-hand asset position of each part by its unit price.

Table 1 (and column one of Summary Table 5)[10] shows the dollar value of delivered airframe spare parts condemned by Technical Orders in the F-94 series of aircraft. Less than 25 percent of the design changes specified in T.O.'s resulted in the condemnation of delivered spares;[11] and comparatively few spare parts actually were condemned.[12] Furthermore, less than 30 percent of the condemned spares had a unit price of more than ten dollars. Only one T.O. lists a spare with a unit price of more than $500 to be condemned. Most of the condemned items were bits and pieces connected with the relatively expensive items that had been affected by the design change. The very sharp increase in the value of condemned parts in 1955 is due to one T.O. which involved a major modification of the F-94.

Spares Modification Costs

The costs of changing delivered spare parts to a new configuration include the dollar value of components and labor used in the modification. The total value of the component ("retrofit") kits purchased by the Air Force to modify delivered spares is obtained by multiplying the cost of the parts contained in the kit by the number of kits purchased. Labor cost

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[9]As of the month in which the data were collected (June 1956), these 151 Tech Orders were all the Tech Orders issued to modify the F-94 aircraft.

[10]The difference between the Table 1 - 4 and their corresponding columns in Table 5 is that the latter cumulates the data shown in Table 1 - 4.

[11]At least another 15 T.O.'s list spares to be condemned, but these spares were found to have no asset position. Another seven T.O.'s, showing inexpensive spares to be condemned, are not included in the total because the asset positions of these inexpensive items could not be determined. Several very inexpensive items are omitted due to difficulty in ascertaining their asset positions. However, even if the number of spares condemned as a result of design change were actually doubled, the derived figure would not alter the picture considerably; very few spares were condemned on the T.O.'s.

[12]Of these 78 condemned part numbers, four had more than a single application per plane.

TABLE 1

Number and Value of Condemned Spares, F-94 Aircraft, by Year

Year	Number of T.O.'s Showing Condemned Spares	Number of Condemned Spares	Dollar Value of Condemned Parts	Percent of Total
1951	1	4	1,229	.12
1952	2	4	290	.03
1953	5	11	3,461	.35
1954	11	29	26,575	2.66
1955	9	32	967,669	96.79
Through June 1956	1	1	87	.01
No Date	1	1	418	.04
	30	78	999,729	100.00

of modifying spares was obtained by multiplying the labor hours involved (estimated on the T.O.) by the number of kits times the cost per labor maintenance hour (estimated at $5.75).[13]

Approximately 60 percent of the 151 Technical Orders call for modifying delivered spares in some way. The total cost of performing these modifications was approximately $600,000 of which about one-third was for retrofit kits and two-thirds for labor. The dollar value of spares modification cost is allocated by year in Table 2 (and column 2 of Table 5).

TABLE 2

Materiel and Labor Costs of Modifying Spares, F-94 Model, by Year

Year	Number of* Tech Orders	Dollar Value of Kits for Spares	Dollar Value of Labor for Spares	Total	% of Total
1951	1	$ 5,940	$ 12,765	$ 18,705	3.02
1952	2	173	1,282	1,455	.23
1953	5	1,554	2,260	3,814	.62
1954	53	126,560	273,177	399,738	64.55
1955	23	84,749	102,051	186,800	30.16
No Date	6	904	7,866	8,770	1.42
	90	219,880	399,401	619,282	100.00

*This list includes only those Tech Orders which show some spares modification costs.

[13]This figure was estimated by the RAND Cost Department in connection with another study. In addition to direct personnel costs, this estimate is meant to include training costs, base overhead, and investment costs.

Contractor Design Termination Costs

Design Termination Costs include spare parts material or work in process that must be reworked or discarded as a result of a design change. Estimates are based on Lockheed records covering design change termination costs, and the scrap and rework costs of inventory material.

When a design change occurs, a Termination Design Change Cost Account (TDC) is set up, and the expenses of rework and obsolescence arising from a reduction in parts ordered are accumulated. These costs cover both production and spare parts; therefore, to isolate TDC charges associated with spares, the total TDC costs were multiplied by the ratio of airframe spares value to total value airframe plus spares. For the F-94 this ratio was 24.1 percent of the TDC costs.

A TDC cost account is held open, accruing costs in the years they arise, until all of the expenses arising from the design change are recorded. This may cover a considerable time period. In this study such expenses have been tabulated as they were recorded (i.e., accrued) in each year regardless of the year that the particular TDC account originated.

Concerning the inventory material scrap and rework cost, the contractor felt that most of this cost could be attributed to design change, with some unknown percentage traceable to careless handling of material in the shops. No correction was made for this factor. The Lockheed cost system does not segregate material scrap and rework costs between production and spares; therefore, the same percentage factor of 24.1 was used here to separate the spares portion of the material scrappage cost from the production portion.

Table 3 (and column 3 of Table 5) presents the totals of spares termination design change costs and inventory scrap and rework costs for the F-94's by time period.

TABLE 3

Termination Design Change Costs, and Inventory Material* Scrap and Rework Costs, F-94 Spare Parts, by Year

Year	Inventory Material Scrap & Rework Costs (Dollars)	Termination Design Change Costs (Dollars)	Total (Dollars)
1948	$ --	$ 830	$ 830
1949	480	10,940	11,420
1950	8,710	14,065	22,775
1951	39,435	30,500	69,935
1952	56,915	207,820	264,735
1953	108,250	85,655	193,905
1954	101,510	77,150	178,660
1955	690	6,855	7,545
	$315,990	$433,825	$749,805

*Obtained from cost accounting records of the Lockheed Aircraft Corporation, Burbank Division.

Limited Applicability Cost

A design change on a particular aircraft model may make a spare item no longer useable in the new configuration. The item is not obsolete, since it can still be used on other series of the aircraft; but the Air Force supply will be more than needed for the remaining demand potential. In the final disposal of parts, records do not permit separation of this excess resulting from design change. Because this cost could not be measured directly, the upper limits of reduced applicability were determined by finding the full-dollar value of all parts whose use was in any way curtailed by a design change. This method, of course, overstates cost, for many parts would have been used before the end item was phased out of service.

The basic data for estimating this cost were taken from the contractor's "exhibits" or lists showing reductions in spares' quantities on order, by reason for the reduction. Where the indicated cause was design change, Air Force and contractor's records established the remaining applicability of each such spare item. The cost of these affected spares was measured by tabulating the dollar value of all such spares delivered to the Air Force. Four types of applicability limitation were established:

a. Only applicable to specified serial number F-94C;
b. Not applicable to F-94C, but to another type, model or series of aircraft;
c. Applicable to specific serial F-94C aircraft, and also to another type, model, and series of aircraft;
d. No remaining applicability (totally obsolete).

The sum of (a), (b), and (c), above, represent the cost of limited applicability, as found by this "exhibit" method.[14]

The dollar value of airframe parts whose applicability was in some way limited by design change is shown in Table 4 (and in column 4 of Table 5). More than 90 percent of the total cost falls into category "a," applicable to specified serial number F-94C aircraft only. Close to 400 different spare line items had their applicability limited by design change. Of these, about 50 items were expensive assemblies with costs of more than $500 per unit. This group of expensive items accounted for almost 80 percent of the value of spares with limited applicability.

Aggregation of Spares Design Change Impacts

The various elements of design change impact are now aggregated, and cumulated over the six years. These elements, cumulated from Tables 1 - 4, are shown in columns (1) - (4) of Table 5. Columns 5 and 6 of Table 5 present the cumulative dollar value of spares design change cost. The figures in Column 5 exclude the costs of limited applicability; these numbers constitute minimum estimates. Column 6 includes the full value of all airframe parts which

[14] The total value of parts in the (d) category provided an independent estimate of the importance of engineering obsolescence. This method* of estimating design change obsolescence established an upper limit for this cost factor, because the figure obtained is that of spares delivered to the Air Force, not, as in the Technical Order method, (see p. 247, above), of spares on hand at the time of the design change. The scope of the data provided in the Exhibits was not identical to that covered in the Technical Orders. However, when the coverage of the two methods was reconciled, somewhat less than $40,000 of F-94 airframe items were made obsolete by design change via the Exhibit method compared to a $32,000 estimate on the basis of the Technical Order method. Since the Exhibit method produces a known over-estimate of such impacts, these two results were taken to be in basic agreement.

*(which we call the "Exhibit method")

TABLE 4

Dollar Value of Airframe Parts with Applicability Limited by Design Change, F-94 Spares, by Type of Limitation and Year

Year	Type of Limitation*			Total
	a	b	c	
1951	$ 425	$ --	$ --	$ 425
1952	87,730	3,988	387	92,105
1953	968,441	41,937	41,007	1,051,385
1954	507,445	23,972	21,208	552,625
	$1,564,041	$69,897	$62,602	$1,696,540

*a Only applicable to specified serial number F-94C.
b Not applicable to F-94C, but to another type, model or series of aircraft.
c Applicable to specific serial F-94C aircraft, and also to another type, model, and series of aircraft.

have had their applicability limited in some way by a design change; these can therefore be viewed as maximum estimates. The last three columns of Table 5 relate the estimates of spares design change cost to the dollar value of delivered F-94 airframe spares.[15] Column 8 shows the minimum estimate of spares design change cost in percent of the value of delivered spares, while Column 9 shows similar information for the maximum estimate.

It appears that at the end of the first six years of the F-94 program, between 3.7 and 6.3 percent of the value of delivered airframe spares had been made obsolete as a result of design change. Surprisingly, despite the fact that the bulk of design change occurs early in the phase-in of a weapon, the impact on delivered spares appears to be relatively small during this early period.

Contrary to expectations, therefore, the spares cost of design change appears to be lowest during the early phase-in of the weapon. During the first two years of an aircraft program, between one and two percent of the value of delivered spares are affected by design changes. Since the greatest number of engineering changes occur early in the program, one would expect the spares design change impact to fall rather than rise over the life of the weapon. Some of the reasons for the reverse situation are:

1. In general, while airframe changes are more numerous during the early phase-in period, they tend to be relatively minor in scope. Many, if not most, of the early design changes affect expensive major assemblies; however, most cases involve relatively minor modifications rather than condemnation of the assembly itself. Parts condemned by reason of design change tend to consist of low-cost components for such major assemblies. Further, even

[15]Estimated from AMC Budget Program Summaries for the relevant years. These data include only the spares initially purchased while the F-94 was in production. They do not include "follow-on spares," which were less than 10 percent of all F-94 spare parts through 1955.

TABLE 5

Cumulative Costs of Design Change on F-94 Airframe Spares Inventory (Thousands of Dollars)

Year	Col. 1 Cumulative Spares Obsolescence	Col. 2 Cumulative Spares Modification	Col. 3 Cumulative In-Plant Spares Obsolescence	Col. 4 Cumulative Limited Applicability	Col. 5 Total Cols. 1, 2, and 3, Cumulative Cost Excluding Limited Applicability	Col. 6 Total Cols. 1, 2, 3 and 4, Cumulative Costs Including Limited Applicability	Col. 7 Value of Delivered F-94 Airframe Spares (Cumulatively)	Col. 8 Percent of Delivered Spares Affected by Design Change	Col. 9 Percent of Delivered Spares Affected by Design Change
								Col. 5 Divided by Col. 7	Col. 6 Divided by Col. 7
1950			34.8		34.8	34.8	5,811	.6	.6
1951	1.2	18.7	104.9	.4	124.8	125.2	21,816	.6	.6
1952	1.5	20.2	369.6	92.5	391.3	483.8	26,403	1.5	1.8
1953	5.0	24.0	563.6	1143.9	592.6	1736.5	54,415	1.1	3.2
1954	31.6	423.7	742.3	1696.5	1197.6	2894.1	64,290	1.9	4.5
1955	999.7	619.3	749.9	1696.5	2368.9	4065.4	64,290	3.7	6.3

if modification costs per unit are substantial, the fact is that most major airframe items have relatively low expected demand rates; hence, early in the phase-in, few such spares tend to be in the system. As a consequence, total spares modification cost tends to be low at this time.

2. Later in the life of a weapon, major modification programs may occur in order to improve the performance of the aircraft rather than to correct an early design deficiency. For example, an interceptor model may receive extensive modification to take advantage of new air-to-air missile. Or, an aircraft designed in an era of reciprocating engines may undergo major modifications by having jet engines added to its power complement. Apparently, during these major modification programs, the bulk of spares design change impact occur. For the F-94, one Tech Order covering a major modification late in the program accounted for 90 percent of the roughly one million dollar value of spares condemned by reason of design change over a period of five years.[16]

3. Expensive spare parts are not often condemned by a design change. They are ordinarily modified, while the associated bits and pieces are condemned or limited in applicability. Only one item with a unit cost of greater than $500 was condemned by all the F-94 Tech Orders. Similarly, less than 25 items with a unit cost of greater than $500 were limited in their application by design change.

4. Contractors and the Air Force show a good deal of administrative flexibility in reacting to design changes. The large number of reductions in orders for parts affected by design change shows the system's responsiveness in this area. For example, on one F-94C contract, approximately $9.5 million of airframe spares were placed on order. While the aircraft under this contract were in production, approximately $4.1 million of the items originally placed on order were cancelled as a result of design change, and spares associated with the newly designed configuration were produced and delivered. Had these design changes not been caught, about $3.1 million of useless spares would have been delivered to the Air Force.

5. In the case of the F-94, most of the spare parts were delivered by 1953. However, the greatest spares impact of design changes occurred after 1953, because the majority of the costly design changes (in terms of limited applicability and spares condemnations) occurred after 1953. Also, the Air Force policy of concurrent delivery of spares,[17] which ensures that the spares delivered have applicability (at least at the time of delivery) to a certain number of end items, would tend to delay the impact of design change on delivered spare parts.

DESIGN CHANGE IMPACTS ON F-100 AND B-52 SPARES INVENTORY

Information covering selected elements of design change impact was also collected for two other weapons, the F-100 and B-52 aircraft, to serve as a limited check on the results obtained for the F-94. Because both the F-100 and the B-52 are relatively new weapons, spares design change costs for only the first two years of the phase-in could be gathered and only two elements, engineering obsolescence and spares modification, could be studied. The results, therefore, are restricted to the early phase-in period and neglect the impact of both in-plant or contractor obsolescence and limited applicability. The data used for estimation consisted of the Technical Orders written on the F-100 and B-52 up to May 1956. The method of analysis was the same as that followed on the F-94 (pp. 246-248).

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[16]The purpose of this T.O. was to improve the cold weather performance of the F-94. The change essentially consisted of a new de-icing system for the aircraft, making components for the old system obsolete.

[17]See footnote 6 on page 245.

Spares Engineering Obsolescence Costs

As of May 1956, 57 Tech Orders had been issued covering F-100 airframe design changes. Of these, 13 called for the condemnation of one or more delivered airframe spares. The dollar value (unit price × quantity on hand) of such condemned spares was approximately $17,500. As in the case of the F-94, almost all of the condemned parts were inexpensive (unit price less than $10). By March 1956, slightly more than $30,000,000 worth of F-100 airframe spares had been delivered to the Air Force.[18] Thus, less than 0.1 percent of the value of delivered airframe spares had been made obsolete by reason of design change during the first two years of the F-100 program. The F-100 program is still too embryonic to make any sweeping generalizations about the ultimate importance of spare engineering obsolescence; but the results obtained appear to corroborate the estimates of design change impact for the F-94 during the early phase-in period.

The importance of spares engineering obsolescence in the B-52 program to date parallels that for both the F-100 and F-94. Of the 256 B-52 Tech Orders issued by July 1956, 36 made some delivered spares totally obsolete. The total dollar value of the roughly 150 condemned parts (as of the most approximate date following the T.O.) came to $133,000. By May of 1956, about 0.3 percent of the $44 million of delivered B-52 airframe items had been made obsolete by reason of design change.[19] Again, the low value of spares engineering obsolescence during the early B-52 program bore out the F-94 results.

Spares Modification Costs

Only eight of the 57 F-100 Technical Orders called for modification of delivered spares. The cost of parts and labor required to perform the necessary modifications totaled $34,000, of which almost 90 percent represented labor expense. Thus, the cost of spares modification came to about 0.1 percent of the value of F-100 airframe spares delivered during the first two years of the F-100 program.

Spares modification was called for on 35 of the 256 B-52 Tech Orders. The value of material and labor required to perform the modification comes to $267,000, of which about 60 percent was for modification kits and 40 percent for labor. As of mid-1956, B-52 spares modification costs amounted to about 0.6 percent of the value of delivered airframe spares.

During the first two years of the F-100 and B-52 program, the cost of spares modification required by design change was less than one percent of the value of delivered spares. These results are consistent with the estimates derived for the F-94 during the early phase-in period.

DISCUSSION OF RESULTS

The cost estimates of spares design changes developed in this study should be interpreted as approximate values. A variety of problems, including changing definitions of key words in Air Force records over time and gaps in available data, made precise results impossible to obtain, but this should not negate the general conclusions drawn from the estimates. While it is impossible to evaluate, precisely, how much has been missed due to gaps in the data, experience with the available data indicate it is highly unlikely that the obsolescence costs under consideration have been underestimated by more than 10 percent. The data were

[18] Obtained from the Spares Administration Division, North American Aviation Corporation.

[19] The value of B-52 airframe assets was obtained from USAF Property Class Management Digest, Directorate Supply and Services, OCAMA.

such that we could at least know where the gaps existed and also what kinds of design changes were involved. In no case is there a data gap involving the impact of a major design change.

In considering the results, the fact that the study examines only one weapon, the F-94, over its first line operational life should be kept in mind. Data on engineering changes indicate, however, that most aircraft behave in basically the same fashion with respect to the timing and the number of engineering changes. Furthermore, the analysis of the design change impact upon the first years of spares support for the B-52 and F-100 weapons support the estimates obtained for the F-94.

In evaluating these results, the fact that the present study estimates the spares impact of design change as it occurs under present Air Force provisioning and procurement policies should be kept in mind. For example, were all aircraft designs frozen for a given weapon, when a specific number of that weapon had been produced, the results would have been different. Also, were the Air Force to spend relatively more resources on design changes of older model aircraft and less in producing new models, in the future, the results of this study would not be immediately applicable to the future. However, no such institutional changes appear in the offing.

The broad conclusion of this study, then, can be stated: over the life of a weapon the cost of airframe spares design change, including obsolescence and modification of delivered spares, in-plant obsolescence, and limited use may eventually be as much as 10 percent of the value of delivered spares. For the F-94 model, a spares cost of five to seven percent appears more nearly correct.

It is not possible to state, with assurance, what provisioning policies are "optimal" in the light of such obsolescence costs. Other costs, such as reordering and warehousing, must also be considered. However, in the light of this study's introductory comments, it appears that the value of purchasing expensive spare parts on a "hand-to-mouth" basis until the design configuration of the end article becomes hardened might be considerable. Expensive (having a unit cost of more than $500) spare parts, which consist of only two percent of the Air Force line items but more than 50 percent of total spares' dollar value, are beginning to be controlled under an Air Force Hi-Valu program. In this "hi-valu" area, the gain that would result from a decrease in spares' obsolescence costs might be considerable, at a relatively small cost. However, the obsolescence costs indicated by this study do not seem to warrant, by themselves, a change in the present procurement policy for "low-valu" items, for which the Air Force is tending toward large (possibly life-of-type) initial buys. For "low-valu" items, the costs of reordering and handling probably are predominant. These are, of course, only tentative conclusions, but it is hoped that the obsolescence estimates supplied by this study will be helpful in reaching a firm conclusion regarding provisioning policies.

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THE DUAL SIMPLEX ALGORITHM FOR BOUNDED VARIABLES*

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I. INTRODUCTION

Applications of linear programming models in which the variables are bounded from above are becoming more frequent [2,5,8,10]. Both Charnes and Lemke [4] and Dantzig [7] have offered a variant of the simplex method for solving the problem

$$\begin{aligned} A\lambda &\leq b \\ 0 \leq \lambda &\leq u \\ \max\ & c'\lambda\ , \end{aligned} \tag{1}$$

where A is an m-by-n matrix, u is an n-by-1 vector of upper bounds on the variables λ, and each relation is an inequality. The entire set of restrictions can, of course, be represented in a linear programming tableau of m+n equations of the form (2) and (3) below and can be solved by regular methods. The aforementioned authors propose a method which utilizes a tableau containing only the first m relations, where at each iteration proper attention is paid to the bounding restrictions. (For the purpose of overall simplicity in presentation, we shall assume that every variable is bounded, perhaps by some "arbitrarily large number" for certain variables; where the number of actual bounding conditions is relatively small, it is probably better to convert these few restrictions into equations of the form (3) below, and follow the usual methods of solution.) Although the technique previously offered entails considerable computational savings over the approach of operating with an m+n equation model, it has two drawbacks of its own:

1. If the vector b is not comprised of entirely nonnegative elements, it may be necessary to obtain a feasible solution by introducing artificial unit vectors [3] with a high cost, or by performing a Phase I [12] calculation employing the bounded variables algorithm. Both devices tend to increase the number of iterations.

2. The suggested algorithm is not simple. Although the criterion selecting the nonbasic variable to increase remains the same (viz., that one associated with the most negative $z_j - c_j$ [3]), the criterion for deciding which vector, if any, leaves the current basis is more

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involved, conceptually and computationally. Three numbers must be compared: the level at which the new vector can be entered into the basis, paying attention only to nonnegativity constraints; the level to which the new vector can be increased, paying attention only to the upper bounds for the current basic variables; and the upper bound level for the new variable. The second level is found by a computation analogous to that for the first level (the minimum of the ratios of the "deficiency" of the i-th basic variable from its upper bound, to x_{ik}, for $x_{ik} < 0$ [3; p. 12]). The rules for altering the tableau* at each iteration depend on which of the three levels is minimal.

As we shall show, Lemke's dual simplex algorithm [9,1,6,11,13] can also be extended to an m equation system with bounded variables, and the modification of his method avoids the above drawbacks. Specifically:

1. An initial feasible solution is always available without the introduction of artificial vectors.

2. Only a minor revision needs to be made in the rules for the dual algorithm.

3. As a corollary to the other reasons, possibly an efficient method for solving ordinary linear programming problems in which the variables are not bounded, but for which it is difficult to find a feasible starting solution for either the regular simplex or dual simplex algorithm, is to "pretend" that the variables are bounded by some arbitrarily large but specified number U* and then apply the algorithm below.

II. DUAL ALGORITHM FOR BOUNDED VARIABLES

The system (1) is converted to a set of linear equations by the addition of m "slack" variables [3] in the first m equations, and n "slack" variables in the upper bound relations

$$\sum_{i=1}^{n+m} \lambda_i P_i = b \tag{2}$$

$$\lambda_i + \lambda_i' = u_i \qquad i = 1, 2, \ldots, n \tag{3}$$

where P_i, for $i = 1, 2, \ldots, n$, are the columns of A, and for $i = n+1, n+2, \ldots, n+m$, are slack unit vectors, and u_i is the upper bound on the i-th variable. We shall refer to the variables in (3) as the i-th pair of complementary variables. If the variable λ_i has a strictly positive coefficient c_i in the optimizing form

$$f(\lambda) = \sum_{i=1}^{n} c_i \lambda_i = z, \tag{4}$$

then substitute λ_i' from (3) into (2) and (4). For example, suppose the first j variables have $c_i > 0$, $i = 1, 2, \ldots, j$; then we write (2) and (4) as

*We shall present our discussion in terms of the original simplex method [3], although the reader should note there are obvious parallel statements which can be made for the revised simplex method [14].

$$(2') \qquad \sum_{i=1}^{j} (-P_i)\,\lambda_i' + \sum_{i=j+1}^{n+m} \lambda_i P_i = b - \sum_{i=1}^{j} P_i u_i$$

$$(4') \qquad f(\lambda) = \sum_{i=1}^{j} (-c_i)\,\lambda_i' + \sum_{i=j+1}^{n} c_i \lambda_i + \sum_{i=1}^{j} c_i u_i .$$

To set up our initial simplex tableau [3], we enter the coefficients specified in (2') and (4'). That is, for every variable in (4) having $c_i > 0$, we record $(-P_i)$, $(-c_i)$ and attach the correct label λ_i' to the column. The entries for variables with $c_i \leq 0$ are recorded in the usual manner. We enter the right-hand side of (2') under P_0 [3]; in general this vector will not be nonnegative. The initial basis is the set of m slack vectors, P_i, $i = n+1, n+2, \ldots, n+m$. Since the right-hand side of (2') may have some negative values, the corresponding slack variables will be negative, and afortiori infeasible. The values $z_j - c_j$ are next computed; since c_j for the slack vectors are zero, and all the other entered c_j in the tableau are nonpositive, $z_j - c_j \geq 0$ for all vectors. Consequently we are ready to institute the dual simplex algorithm.

Step 1. Criterion for a variable to leave the basis. This criterion remains unchanged, viz., the most negative variable in the basis, say the r-th, is to be removed. If all basic variables are nonnegative, an optimal solution has been found as we shall indicate below.

Step 2. Criterion for a variable to enter the basis. This criterion is also unchanged, viz., find the minimum ratio of $(z_j - c_j)/-x_{rj}$, for all j such that $x_{rj} < 0$; where this minimum occurs determines the entering variable.*

Step 3. Tentative new tableau. Since the vector to be removed and the vector to enter the basis are ascertained, the usual transformation of one tableau to the next is carried out.

Step 4. Finalized new tableau. For the first time in the algorithm (aside from the initializing procedure) we consider the upper bounds. Note that (3) defines an upper bound for both λ_i and λ_i'. If any basic variable (λ_i or λ_i'), say the t-th, is above its upper bound, then (1) the sign of all the elements in row t of the tableau are changed except for the coefficient 1 appearing in the column corresponding to this basic variable; (2) the value of the t-th variable in the P_0 column is replaced by the value of its complementary variable from (3);† (3) the label for the t-th variable is replaced by the label of its complementary variable; and finally, (4) the sign of c_j for the t-th variable is changed.‡

In short, Step 4 consists of a set of sign changes, relabelings, and trivial numerical substitutions as defined by (3). Mathematically, all we have done is to replace the t-th

*We shall not discuss here the resolution of ties and the associated degeneracy question; see [1,6,9]. If all $x_{rj} \geq 0$, no feasible solution exists for the entire model.

†This quantity is also needed at each iteration of the primal algorithm for bounded variables.

‡At any iteration, either λ_i or λ_i' appears in the tableau, but never both. The indicated alterations leave $z_j - c_j$ unchanged.

variable by its complementary variable and make the necessary alterations in the simplex tableau; since the t-th variable exceeded its upper bound, the complementary variable is negative, demonstrating the current infeasibility.

Return to Step 1.

III. DISCUSSION

At the terminating iteration there are four mutually exclusive and completely exhaustive classifications of variables (footnote ‡, p. 259): (1) nonbasic λ_i; (2) nonbasic λ_i'; (3) basic λ_i, and (4) basic λ_i'. Categories (1) and (3) indicate optimal values for these variables. Category (2) implies that $\lambda_i = u_i$, and category (4) indicates that $\lambda_i = u_i - \lambda_i'$ in the associated optimal solution.

The proof of feasibility and optimality of the final nonnegative basic solution obtained from this algorithm is exactly analogous to the proofs of Charnes and Lemke, and Dantzig. Feasibility of the obtained solution with respect to the upper bounds is assured by the nonnegativity of the basic variables and Step 4. Feasibility in the first m equations is assured by noting that Steps 1 through 3 comprise the ordinary duel simplex algorithm whose correctness has been established by Lemke [9], and that Step 4 consists of merely a change of variables according to (3). Finally, optimality can be demonstrated by noting that (3) actually represents a system of "secondary constraints" [7], that the final basic solution is feasible and optimal for the "primary constraints," and since it is also feasible in the secondary constraints, it is optimal for the entire system. Similarly, the proof of convergence of the method is nearly identical to that for the unmodified dual simplex algorithm.

We remarked earlier that the present algorithm may provide an efficient procedure for any linear programming problem in which a nonartificial starting basis is difficult to find. Often by inspecting certain of the first m restrictions or by referring to previously found optimal programs for similar models, reasonable upper bounds can be established for variables with $c_i > 0$. The "starting procedure" above may be employed to provide an initial feasible solution for the dual algorithm. If the upper bounds selected are truly ineffective, Step 4 of the procedure is never used, and the iterations proceed according to the standard dual simplex algorithm.

In closing, we point out that the dual algorithm for bounded variables has three potential drawbacks, viz., if no feasible solution to the entire model does exist, then it may take more iterations by the present method than by the two-phase approach [12] to discover this condition; if a large number of variables have $c_i > 0$, and most of these do not appear at their upper bound in an optimal solution, under the present approach it will take at least one iteration to remove the variables from their upper bound, i.e., to undo the transformation required in the starting procedure; the dual simplex algorithm may be more difficult to code for an automatic computer. In our opinion, these difficulties will prove to be minor and will be substantially outweighed by the relative computational simplicity of the dual algorithm over the modified procedure for the regular simplex method.

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THE ALLOCATION OF DEVELOPMENT FUNDS: AN ANALYTIC APPROACH*

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The administrative problem of fund allocation among the several projects constituting the development program is considered from an analytic-economic viewpoint. The concept of operational potential is introduced and used in establishing a measure of effectiveness for the development program, the "development contribution function." The problem then resolves to the maximization of a mathematical expression. An example is worked out to demonstrate the method and the utility of the results. Certain characteristics of the solution which are of general applicability are discussed. The functional relationships among the groups concerned with development is considered; the contribution of each group is defined in the form of equation parameters. The importance of well designed development administrative machinery, particularly relating to the integration of the operational and the technical elements, becomes immediately apparent.

INTRODUCTION

The activity generally referred to as "development" properly falls within the realm of logistics because it deals with the provision of the wherewithal to operate effectively. In the prosecution of a development program certain difficult and unique administrative problems are raised, whose solution—or lack of solution—will affect the quality of the products and hence the success of operations. Unlike the more usual logistic functions concerned with the provision of existing material (distribution) or the manufacture of standard items (production), development creates where nothing existed before. It is the process of transformation from idea to reality. The difficulty arises from the fact that this transformation process has no equilibrium state. Each problem is sufficiently novel that generalizing even the method of solution becomes dangerous. It is very difficult to formulate a development doctrine which is both sound and universally applicable.

The question of fund allocation is an economic problem involving the optimization of some quantity. If this were not so, then any distribution would be equally good and there would be no problem. Before we can answer whether we're getting the most for our money, we must consider what it is we are buying. It is necessary, therefore, first to describe the market we are dealing in to define our usage of the term "development."

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An organization undertakes a program of development to enhance its ability to execute its mission. As a measure of this ability, the concept of operational potential is introduced. Operational potential is a function of time, place, and circumstance; its value in a given situation will depend on the objective, own forces, forces opposed, environmental factors, and the other classical variables. Development is concerned with the future quality of "own forces;" its success or failure is to be judged by the degree to which it ultimately strengthens their capabilities in terms of operational potential.

The close relationship of development to operations leads then to the following definition:

> Any form of creative technical activity which promises a direct and predictable contribution to operational potential is classed as development.

It is the intent of this operational definition to distinguish between development and research. The words "direct" and "predictable" are essential modifiers of development activity. Within the meaning of this definition, if the operational use of the object or study is not clear then it is not development. It is necessary in following the rest of the argument that this differentiation be kept clearly in mind; for nothing is asserted about research fund allocation. It is worth noting that although the majority of development items may be comprised of hardware, the field is certainly not limited to tangible things. For example, analysis whose objective is the creation of new tactics or operational procedures would be classed as development within the scope of this definition.

Money spent on development does not buy any return until a particular project is completed and its results placed in service. At this time, and not before, it can be said that the developmental money has been traded for improved capability of the operating forces. Consider, for instance, a project whose aim is the creation of a new weapon. When the new weapon is placed in service, operational potential increases by a discrete jump; on the other hand, the half developed weapon is of no operational value and contributes nothing to operational potential. But certainly the partially developed weapon represents a degree of accomplishment and has contributed to future security, for the time has been reduced to the day when the new weapon shall be operational. One might say that the development market deals in operational futures through current technology. An evaluation function is sought for the development program which combines the ultimate operational value of its projects with the technical achievement in partially completed, continuing work. The optimization of this function by the proper allocation of project funds will insure that development money is utilized to its full capacity.

The Problem

Suppose the development program consists of N projects, numbered 1 through N. With each a "project operational potential," I_j, is associated. This quantity is a measure of the expected increase in ability of the organization to fulfill its mission on completion of the jth project. For any given period of time, we consider a "project development contribution," A_j, as a function of the project operational potential and the fraction of the project planned to be accomplished during the period, that is

$$A_j = I_j P_j \qquad j = 1, 2, 3, \ldots N \tag{1}$$

where P_j is the "project completion fraction." P_j expresses the expected relative completion of the project; thus $0 \leq P_j \leq 1$. The project completion fraction depends on the nature of the

problem and the approach to a solution, the people and facilities engaged in its solution, the allocated money, and the length of the planning period. Of these only the allocated money is to be considered a variable under administrative control; the remaining factors are to be thought of as already determined and not subject to change.

As time goes on and the military picture unfolds, project operational potential may change, but here it is to be considered a constant. It is assumed further that each project is independent of all other projects, i.e., A_j depends only on the importance of and progress in the jth project and neither of these are affected by any other segment of the program. On summing over all A_j, an evaluation function for the entire development program is obtained, the "development contribution function."

$$A(R) = \sum_{j=1}^{N} A_j(r_j) = \sum_{j=1}^{N} I_j P_j(r_j) \tag{2}$$

where r_j is the money to be allocated to the jth project during the planning period and R is an n-tuple,

$$R = \left|\left| r_1, r_2, \ldots\ldots r_N \right|\right|,$$

representing a point in "fund space" subject to the constraint that the total money available for the program is fixed. That is,

$$c \geq \sum_{j=1}^{N} r_j \qquad r_k \geq 0 \qquad k = 1, 2, \ldots\ldots, N \tag{3}$$

One approach to the fund allocation problem of the development administrator is to maximize (2) subject to the constraint (3).

When A is convex and differentiable, the following conditions on R and (a Lagrange multiplier) λ are necessary and sufficient that R be a solution for the constrained maximum problem above; otherwise, if A is not convex, but differentiable, the conditions are necessary:*

$$\frac{\partial A(R)}{\partial r_k} = I_k \frac{dP_k(r_k)}{dr_k} \leq \lambda \qquad k = 1, 2, \ldots\ldots, N \tag{4}$$

$$\sum_{k=1}^{N} \left[\frac{\partial A(R)}{\partial r_k} - \lambda \right] r_k = \sum_{k=1}^{N} \left[I_k \frac{dP_k(r_k)}{dr_k} - \lambda \right] r_k = 0 \tag{5}$$

*H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," J. Neyman (Ed), Proceedings of the Second Berkeley Symposium on Mathematics, Statistics and Probability, 481-492, U. of Calif. Press, Berkeley 1951. See also, for easier reading without proofs, A. W. Tucker, "Linear and Nonlinear Programming," Operations Research 5:244-257 (1957).

(3) $$\sum_{k=1}^{N} r_k \leq c \qquad r_k \geq 0$$

(6) $$\lambda \geq 0 \ , \ \lambda \left(c - \sum_{k=1}^{N} r_k \right) = 0$$

Since the first and last inequalities require each term of the sum

$$\sum_{k=1}^{N} \left[I_k \frac{dP_k(r_k)}{dr_k} - \lambda \right] r_k$$

to be nonpositive, each term must be zero by the second equation above, thus, these conditions may be rewritten in the form:

for $k = 1, 2, \ldots, N$

(7) $$\text{if } r_k > 0 \text{ , then } I_k \frac{dP_k(r_k)}{dr_k} = \lambda$$

(7') $$\text{if } r_k = 0 \text{ , then } I_k \frac{dP_k(r_k)}{dr_k} \leq \lambda$$

(3) $$\sum_{k=1}^{N} r_k \leq c \ , \ r_k \geq 0$$

(6) $$\lambda \geq 0 \ , \ \lambda \left(c - \sum_{k=1}^{N} r_k \right) = 0 \ .$$

Hence, when $r_k > 0$, for all k, we obtain the ordinary relationship, (7), associated with Lagrange multiplier when no inequality constraints are present.

The r_j which solve (7) are the coordinates of the solution point, R, in fund space. To proceed further requires closer consideration of the project completion fraction, P_j.

Characteristics of the Development Contribution Function

The project cost, say E_j, is the total money required to complete the project. It depends on the rate of expenditure, has a minimum, and becomes very large for very large or very small rates.

If a project is completed at a uniform rate of expenditures, over several time periods, say r_j per period, we have a relationship such as

$$P_j(r_j) = \frac{r_j}{E_j}. \tag{8}$$

If uniform rates of expenditures are employed, then E_j will depend on that rate, r_j. At low rates a greater portion of the monetary effort supports nonproductive overhead and sometimes not enough is available to provide the essentials for progress in the technical work, such as capital equipment, supplies, power, etc. Extremely low rates support only part time productive technical work and result in an inefficient utilization of time. As the rate decreases still further, the condition is eventually reached where nothing but paper-work overhead can be accomplished. Clearly under these conditions the technical work could never be accomplished. At this expenditure rate the total project cost would have no upper limit, i.e., $E_j \to \infty$ as $r_j \to 0$.

As r_j becomes larger a greater proportion of money is spent productively. The effort begins to assume the character of coherence and mutual support. Monetary efficiency increases and the total project cost, E_j, decreases. A point is reached where E_j is a minimum. A large part or all of the available first rate technical talent is committed. The technical problems are being attacked vigorously and competently. Waste is minimized.

As r_j is increased beyond this point the project cost function again starts to increase. The saturation of technical talent requires the employment of poorly trained and unskilled workers. Interference, duplication, and breakdown of communications between groups begin to be seen. Money tends to buy less progress per dollar. The project becomes cumbersome. Finally, as r_j increases without limit, so does E_j, i.e., $E_j \to \infty$ as $r_j \to \infty$.

There are other restrictions imposed on E_j which may be seen on closer examination of the project completion fraction, P_j. By definition, in no case can the value of P_j exceed unity; indeed, P_j may approach a limiting value considerably less than unity. This is to say that in general there exists a maximum rate of progress toward the solution of a technical problem, a limiting velocity in a sense. For example it may be that the solution of certain subproblems must proceed in order; part number one must be solved before part number two can be undertaken, etc. The necessity for such solutions "in series" will tend to limit the rate of progress. In addition, experimental work is full of instances where nature sets certain minimum time limits on yielding up her information, e.g., it may be required to subject a piece of material to a given temperature for a certain interval of time, a week perhaps. All of these factors tend to set an upper limit on the progress that can be made in a given time irrespective of the money available.

On the other hand, the allocation of more money to a project must always increase its completion fraction. One can never get less progress for more effort. This is an asssumption, of course; it is a reasonable one though, and might be called "the assumption of good administration."

These properties of P_j may be stated as follows:

$$\lim_{r_j \to \infty} P_j(r_j) \leq 1 \qquad \text{and} \qquad \frac{dP_j(r_j)}{dr_j} > 0 \qquad r_j > 0.$$

TABLE 1
Conditions on the Functions

	$r_j \to 0$	$r_j = r_{oj}$	$r_j \to \infty$
E_j	$\to \infty$	$= E_{oj}$	$\to \infty$
$\frac{dE_j}{dr_j}$	$\to -\infty$	$= 0$	$\to T_{mj}$
P_j	$\to 0$	$= P_{oj}$	$\to P_{max\,j}$
T_j	$\to \infty$	$= T_{oj}$	$\to T_{mj}$

The "project completion time," T_j, is the reciprocal of P_j. It is the expected number of planning periods required to complete the project.

$$T_j = P_j^{-1}$$

The project cost function has an asymptote whose slope is equal to the minimum completion time. The conditions on the function are summarized in Table 1.

A project cost function which has three independent parameters and satisfies all of the conditions of Table 1 is the hyperbola given by

$$E_j = \frac{T_{mj}\, r_{oj}^2}{r_j} + T_{mj}\, r_j + E_{oj} - 2\, T_{mj}\, r_{oj} \tag{9}$$

where the parameters are:

Minimum Completion Time - T_{mj}
Minimum Cost - E_{oj}
Rate for Minimum Cost - R_{oj}

If the "optimum completion time" is defined as the time required to complete at the rate for minimum cost, i.e.,

$$E_{oj} = r_{oj}\, T_{oj},$$

then (9) may be expressed as follows:

$$E_j = \frac{T_{mj}\, E_{oj}^2}{T_{oj}^2\, r_j} + T_{mj}\, r_j + E_{oj}\left(1 - 2\,\frac{T_{mj}}{T_{oj}}\right). \tag{10}$$

A dimensionless curve is obtained by letting

$$E_j = \frac{E_j}{E_{oj}},$$

$$\rho_j = \frac{r_j}{r_{oj}}$$

and

$$\tau_j = \frac{T_{mj}}{T_{oj}}$$

$$E_j = \frac{\tau_j}{\rho_j} + \tau_j \rho_j + (1 - 2\tau_j) . \tag{11}$$

Then

$$\rho_j = \frac{1}{\tau_{oj}} \frac{\rho_j}{E_j} \quad \text{and} \quad \tau_j = \tau_{oj} \frac{E_j}{\rho_j} . \tag{12}$$

Project cost function, completion fraction, and completion time curves are shown in Figures 1, 2, and 3, respectively.

Using the equation form of the Lagrange multiplier conditions (assuming for the moment, that the solution R is such that $r_k > 0$, for all k), and by substituting relation (8) into the conditions (7), (7'), (3), (6) for a solution, we have, for $k = 1, 2, \ldots, N$

$$\text{if } r_k > 0, \text{ then } \frac{I_k \left(E_k - r_k \frac{dP_k}{dr_k} \right)}{E_k^2} = \lambda , \tag{13}$$

$$\text{if } r_k = 0, \text{ then } \frac{I_k}{E_k} \leq \lambda , \tag{13'}$$

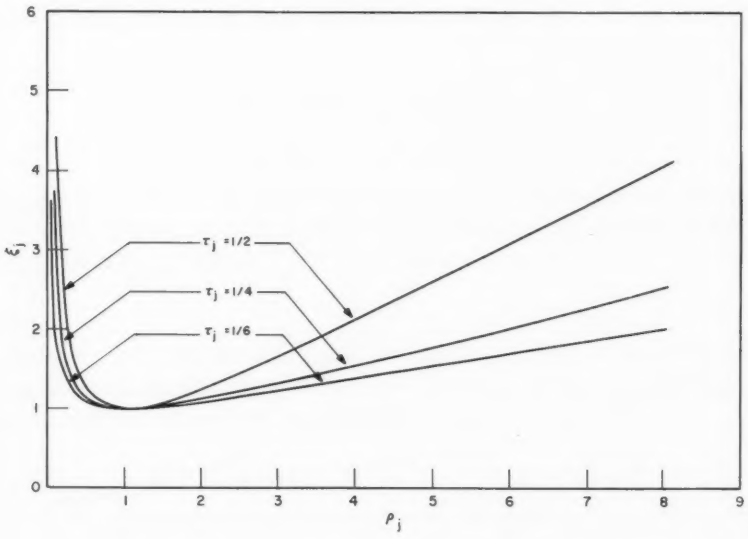

Figure 1 - Project cost functions (dimensionless)

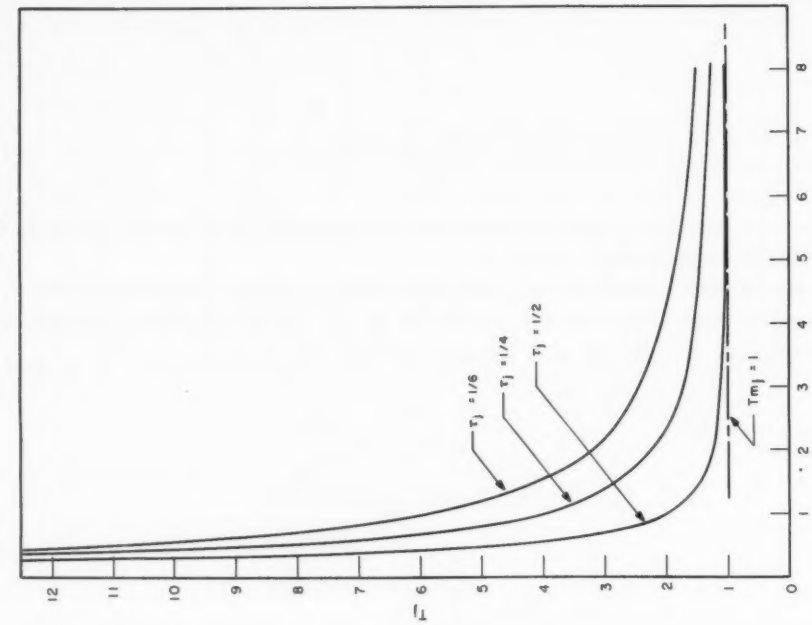

Figure 3 - Project completion times

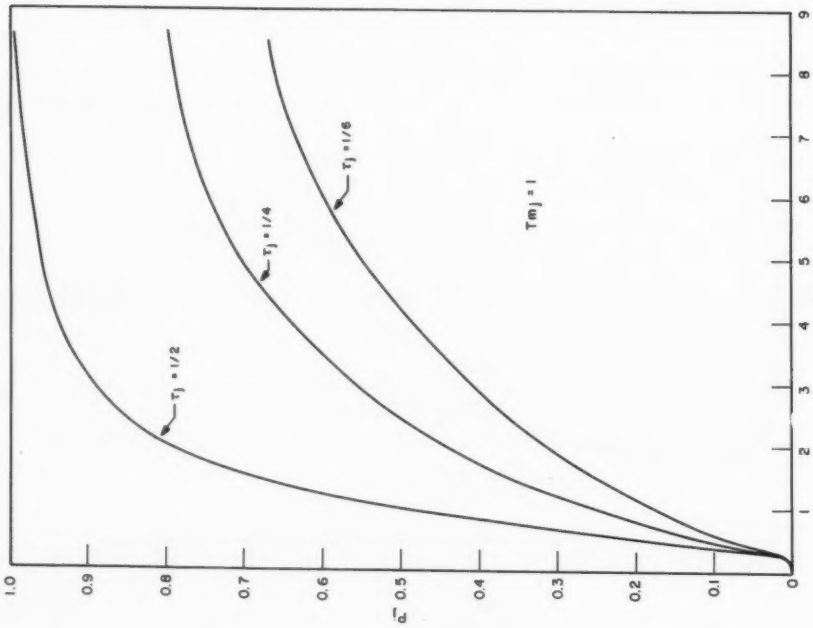

Figure 2 - Project completion fractions

(3) $$\sum_{k=1}^{N} r_k \leq c \quad , \quad r_k \geq 0 ,$$

(6) $$\lambda \geq 0 \quad , \quad \lambda \left(c - \sum_{k=1}^{N} r_k \right) = 0 .$$

If we assume $\frac{I_k}{E_k} > 0$ for all k, then by (13') $\lambda > 0$, and, hence, by (6), we must always have $\sum_{k=1}^{N} r_k = c$. Now, let $K(r) = \{k \,|\, r_k > 0\}$ be the set of "active" projects in an allocation.

Then a solution may be sought by determining a $K < \{1, 2, \ldots, N\}$ and r_k, $k \in K, \lambda$, such that

(14) $$\frac{I_k \left(E_k - r_k \frac{dP_k}{dr_k} \right)}{E_k} = \lambda \quad , \quad k \in K ,$$

(15) $$\sum_{k \in K} r_k = c ,$$

and, in addition, the r_k and λ so found satisfy the relations

(16) $$r_k > 0 \quad , \quad k \in K \quad , \quad \lambda \geq \frac{I_k}{E_k} \quad , \quad k \notin K .$$

Relations (14) and (15) can not be explicitly solved. But in specific problems, "trial" solutions to these relations can be found by graphical methods for various K, and if a solution to the maximum problems exists (16) will be satisfied for some K.

An Example

An example will illustrate the method and demonstrate the utility of the results. The optimum fund distribution will be worked out for a program of four projects spread over a wide range of operational importance and technical difficulty.

TABLE 2

Project No.	1	2	3	4
Minimum cost, E_{oj}:	500	100	500	100
Optimum time, T_{oj}:	5	5	5	5
Minimum time, T_{mj}:	2	2	2	2
Operational potential, I_j:	10	10	1	1

The qualitative distribution of projects in the pertinent parameters is indicated in Table 3.

TABLE 3
Project Distribution

	Technically difficult	Technically routine
Operationally important	Project 1	Project 2
Operationally unimportant	Project 3	Project 4

Using (8) and (10), the constrained maximum problem given by (2) and (3) is,
Maximize

$$\left[\frac{10r_1^2}{2r_1^2 + 100r_1 + 20{,}000} + \frac{10r_2^2}{2r_2^2 - 60r_2 + 800} + \frac{r_3^2}{2r_3^2 + 100r_3^2 + 20{,}000} + \frac{r_4^2}{2r_4^2 - 60r_4 + 800}\right]$$

when

$$r_1 + r_2 + r_3 + r_4 \leq c\,,$$

$$r_1 \geq 0\,,\quad r_2 \geq 0\,,\quad r_3 \geq 0\,,\quad r_4 \geq 0\,.$$

Figure 4 shows the allocations which outline this maximum as a function of c—total available funds. Among the interesting features of these curves, the existence of "thresholds" is most obvious. A project threshold is the minimum total funds required to permit support of a given project. In the example the thresholds are seen to be:

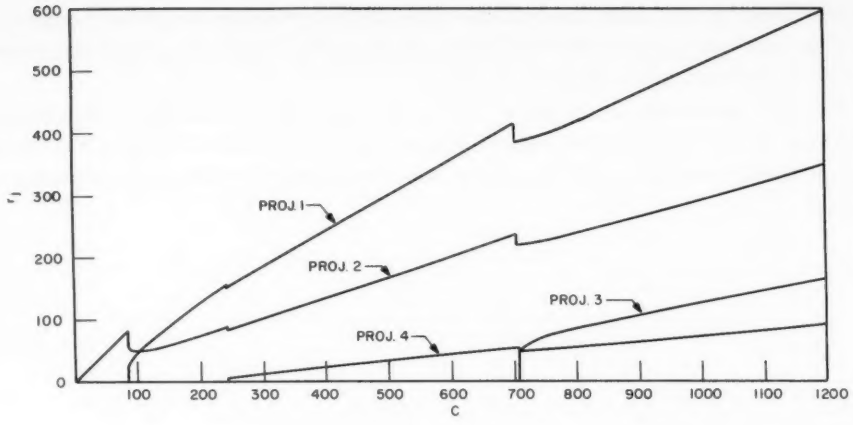

Figure 4 - Allocation functions

Project	Threshold
1	0
2	85
3	242
4	706

At a threshold, discontinuities in project support are apparent. These follow from the fact, inferrable from Figure 1, that it is uneconomical to support certain projects at less than a certain minimum rate.

Project	Minimum Rate
1	26
2	0
3	50
4	6

As a threshold is reached, funds must be drawn from existing projects to support the new one. This is one of the least a priori obvious results of the theory and one of the most difficult to put into practice. When new projects are undertaken, the generally followed procedure is not to diminish existing support on other projects. It is extremely difficult for an individual concerned with a single project to understand why he must accept less money when there is an increase in the total available. But from an overall sense, such a cutback may be in the best interests of the program as a whole, as it may yield the maximum possible increase in total operational potential.

Another feature to be gleaned from Figure 4 is the "crossover" of the functions for projects of equal operational potential. The rule may be stated as follows: in the region very

near threshold the technically simpler of two projects of equal operational potential receives the greater support; well past threshold the more difficult receives the greater support. This behavior is explained by noting that the point of diminishing returns in operational potential is met sooner in the technically simpler project.

Figure 5 shows the percentage of total funds required by each project as a function of total funds available. In the broad regions between threshold points it is seen that the percentages are fairly constant, with the larger ones diminishing slightly and the smaller ones increasing.

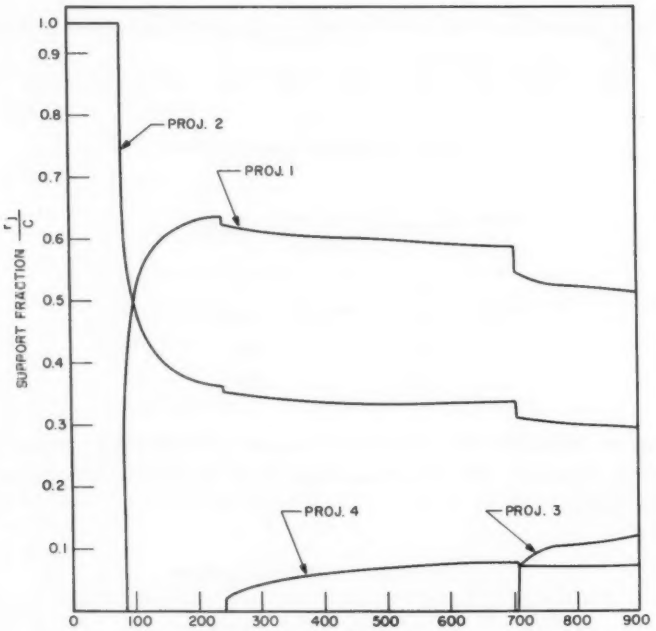

Figure 5 - Support fractions

Figure 6 represents the total operational potential, A, as a function of total funds available, C. As mentioned earlier this function must increase monotonically. Since it must necessarily approach the asymptote, A_1, it follows that a negative curvature is required. This figure represents the maximum possible operational potential available to the program for any given amount of money.

These four ideas—threshold, minimum rate, discontinuity, and crossover—are results of this type of analysis which are by no means self evident in a nonanalytic approach. Furthermore, the quantity of useful information contained in these results is quite large. Not only is the optimum fund distribution determined for a particular value of total program funding, C, but continuously as a function of C. This information (Fig. 4) together with the increase in total operational potential expressed as a function of C (Fig. 6) suggest several means of

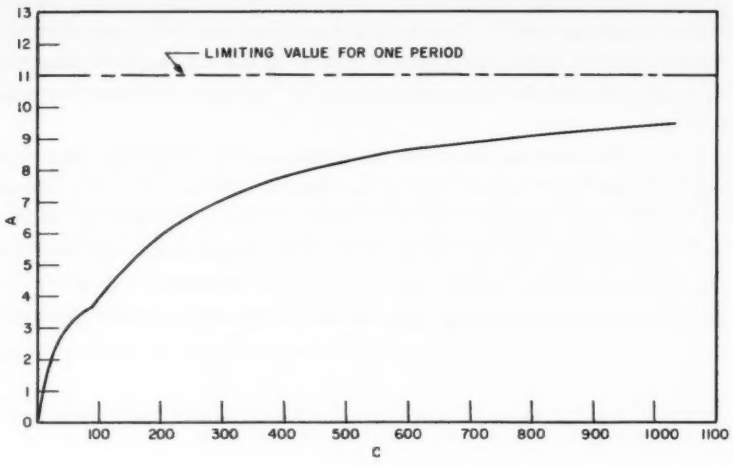

Figure 6 - Total operational potential

determining what the total program effort should be. One such method might begin with the specification of a minimum acceptable increase in total operational potential, A, on the basis of the projected military situation. A curve similar to Figure 6 would then find a value for C.

DISCUSSION

The methods presented in this paper are not to be considered as a substitute for the judgment of the administrator. They are to be thought of more precisely as tools to aid the exercise of judgment by the provision of a systematic quantized basis for its operation. Just as sextant, compass, and chart do not replace the navigator's judgment but tend to reduce the area of guess and hunch, leaving his judgment free for the crucial decisions, so this method is intended to provide a basis for the systematic exercise of administrative judgment on the important questions. The ultimate goal is sound executive decision in the field of development.

At the high administrative level the development program must be considered as a unified whole rather than a loose collection of independent parts, each demanding and receiving its due without regard for the total resources available. The development contribution function (2) presents such a view.

It is generally recognized that development administration requires the coordinated efforts of several organizational groups, including the operational, technical, and fiscal. Of particular importance is the integration into the administrative machinery of the senior operational command where decisions relative to the future commitments of the organization are made. The analytic approach defines the form of this coordinated effort; it specifies the contribution of each echelon in the form of equation parameters. The determination of project operational potentials is quite naturally a function of operations. This responsibility, as well as that for the generation of realistic operational requirements, implies a familiarity on the part of operations with the expected performance capabilities of the various developmental projects and an appreciation of the capabilities and limits of technology in general. A degree of technical competence is thus required of command.

Conversely, the technical level must be kept advised of operational requirements in order to generate suitable projects. There should be clear and well used information channels between all groups involved in the development effort. If the development administrative machine is to produce effectively, there should also be "positive feedback" between and among its various elements.

Equation (2) is the meeting place for the collaboration of operator and technician. The rules of the game are specified in the subsequent development to (14), (15), (16) and the optimum fund allocation distribution (Fig. 4). For an effective coordinated effort each group should not only know the rules and its own part, but also the problems and functions of the other groups as well. There is a very definite requirement for technically competent operators, technicians with operational experience, and administrators well grounded in both.

* * *

NOTES

PROBLEMS

It has been suggested that this journal might serve as a medium for publishing problems in the area of Logistics — with the idea that other persons might be interested in those problems and might submit comments. Readers are invited to submit brief statements on applied and theoretical problems in Logistics. Address letters to Managing Editor, Naval Research Logistics Quarterly, Office of Naval Research, Washington 25, D. C.

NEWS AND MEMORANDA

Readers are invited to submit to the Managing Editor items of general interest in the field of logistics

Movie on Logistics System Simulation

The Air Force has recently completed a film which describes the first experiment in simulation conducted by the RAND Logistics Systems Laboratory (short title: LP-I). This experiment dealt with the testing of a series of new supply policies to be used for the management of spare parts used on USAF aircraft. These policies covered procurement, repair scheduling, and distribution of airframe spare parts, and the simulation consisted of using men and machines to conduct the experiment. The machines, particularly a large scale electronic computer, represented the physical activities and routine decisions arising in a supply system. The men simulated key logistics managers who provided the important decisions needed and who monitored the routine decision-making performed by the lower level personnel as they were simulated in the computer.

The film describes the policies, the simulation technique used, and tries to convey the general atmosphere in which the participants worked during the experiment. The movie is 30 minutes in length, and is in black and white. The Laboratory staff and the experimental participants provided the "actors" for the movie. The Air Force motion picture staff wrote the script and produced the movie. RAND provided technical assistance.

The film is identified as "Simulogs" (SFP532). Inquiries concerning the film should be made to the RAND Corporation, 1700 Main Street, Santa Monica, California.

Interdisciplinary Conference on Self-Organizing Systems - Chicago, Illinois

An Interdisciplinary Conference on Self-Organizing Systems will be held on May 5 and 6, 1959, in Chicago, Illinois. The conference is being sponsored by the Office of Naval Research, Information Systems Branch, with the assistance of the Armour Research Foundation of the Illinois Institute of Technology. The major function of this conference is to bring together research workers in the fields of the Biological, Mathematical, Physical, Psychological, and Social Sciences who have actively considered the development and growth of Systems capable of

spontaneous classification of their environments. Emphasis will be placed primarily on non-biological mechanisms which exhibit characteristics of organization and growth, of thought and learning, of information and communication – those characteristics which are normally attributed to intelligent organisms.

Interested individuals may obtain further information and a preliminary program, when available, by writing to Mr. Scott Cameron, ICSOS Conference Secretary, Armour Foundation, 10 West 35th Street, Chicago, Illinois.

RECENT PUBLICATIONS

EDITOR'S NOTE: Listing of a publication in this section is for record and reference only and does not constitute an endorsement of point of view or advocacy of use.

SCIENTIFIC PROGRAMMING IN BUSINESS AND INDUSTRY. By Andrew Vazsonyi. John Wiley and Sons, Inc., New York, 1958. 474 + xix pp. $13.50.

Dr. Vazsonyi intends his book as an excursion in business management, not mathematics. Some of the mathematical techniques which are gradually winning favor in industry are carefully discussed for the benefit of businessmen "willing to spend the time to assimilate the meaning and potentialities of these new scientific tools." The book is a model of clarity and simplicity; but the intended reader may not realize this until he has become accustomed to interpreting frequent mathematical symbols and equations in the text, and working through the numerical examples. No formal mathematical background is assumed for the reader; only occasionally has it been necessary to use simple integration or differentiation.

It would seem from the opening chapter that "scientific programming" might as well be called "operations research," or any other of the names attached to use of the techniques for decision making and problem solving discussed in this and other journals. Certain problems from these journals are presented as demonstration of achievement through use of these techniques. We may divide the book into four parts of unequal length, each part covering a topic or technique from the literature of the past few years. These are: mathematical programming (linear, quadratic, and dynamic); game theory; production and inventory control; and production scheduling of assembly lines and job shops.

The coverage of linear programming should broaden the understanding of this subject, for here is a thorough and clear discussion in 174 pages. The transportation problem is dealt with first, then the simplex solution. Implications of duality are presented; the procedure of assigning row and column numbers to the transportation matrix is introduced as a computational shortcut. The "generalized transportation method" is introduced, and illustrated by the aircraft routing problem of Ferguson and Dantzig. (This is the pattern of presentation through most of the book - problems drawn from the literature of the past eight years are "solved" by applying the appropriate technique.)

A chapter is devoted to the geometry of linear programming. The three-dimensional perspectives of some unsung draftsman deserve special mention. These are most helpful in clarifying the problem solved by linear programming, and in establishing for the reader the relation to game theory.

Degeneracy is briefly considered in the discussions of the algorithms (the perturbation method is recommended should the problem arise in computation) and the geometric interpretation is drawn.

A careful reading of this presentation will benefit the beginning student of linear programming, since it provides an understanding that is an excellent basis for study of more advanced literature. In light of the emphasis on application, additional space might have been devoted to further discussion of computational shortcuts; Kuhn's Hungarian method is not treated, though the allocation problem is considered; and no mention is found of techniques for arriving at advantageous initial allocations, such as the Vogel Approximation Method. Orden's transhipment modification, a useful form that should find wide application, also is not mentioned.

Two chapters are devoted to brief introductory expositons of dynamic programming; and of "convex programming," which describes the work of Charnes and Lemke on the minimization of non-linear separable convex functionals.

Game theory has not yet been extensively applied to business management; this is reflected in the chapter devoted to this subject, which derives its interest from mathematical ideas, not business ideas. In the light of the book's purpose, this may be its weakest chapter. It has been a standard practice in the past to present the "Colonel Blotto" problem as an analogy to opposition of interest in a business situation, and we may report gratefully that the author does not continue this practice. Instead, an interesting problem of planning the output of a refinery while considering the behavior of a competitor is considered in the format of a two-person zero-sum game. Once the problem is posed, the discussion proceeds along familiar lines; strategies are identified and evaluated for a Morra fingertossing game. The reader is left to apply the conclusions to the business problem. The presentation might also have benefited from a mention of implications of work accomplished and in progress beyond the two-person zero-sum level.

A long chapter is devoted to methods for controlling inventory and scheduling production when demand is uncertain. A brief presentation of some probability concepts is followed by expositions of the models discussed by Whitin, Arrow-Harris-Marschak, and Holt-Modigliani-Simon; and variations are proposed for handling special problems, e.g., Planning Production of a large number of parts. Minimization of a loss function under various conditions is clearly presented, and a method for imputing shortage costs from the control policy is made available. A word of caution advising the reader that lot size formulae cannot be used for production scheduling except under special circumstances would have been in place. And the presentation of the basic probability concepts is too cursory, in the opinion of the reviewer. When supplemented by some background such as is found in Allendoerfer and Oakley: Principles of Mathematics (New York, McGraw-Hill, 1955), or in Moroney: Facts From Figures (Baltimore, Penguin Books, 1956), the chapter should serve well as an introduction to its subject.

Three final chapters discuss systems for scheduling assembly lines and machine shops. In an area where no method exists for obtaining optimal solutions except in the very simplest cases, the Gantt charts and other graphical devices explained here are useful in systematic scheduling.

There are many typographical errors, some of which may confuse or mislead the reader. A bibliography would have been welcome, particularly if classified, as was Bross' in Design for Decision (New York, Macmillan, 1953), according to the level of the mathematical content. The book makes more accessible some of the major achievements of operations research without simplifying these achievements to meaninglessness. It deserves high recommendation as an introductory text. Unfortunately, the price may deter college students interested in the subject matter, for the book should prove useful to this group, also.

(Reviewed by Daniel Orr)

FUNDAMENTALS OF MATHEMATICS (Revised Edition) 1958. By Moses Richardson Published by: The MacMillan Company, 507 + xviii pages. $6.50

This is a book which is designed to afford a better understanding of mathematics to the non-mathematician. In the Preface to this Revised Edition the author states:

> "The book attempts to provide a suitable terminal course for students of the arts and social sciences, stressing the fundamental concepts and applications of mathematics rather than its formal techniques. However, considerations of this kind would also be beneficial for students of the sciences and prospective teachers of secondary school mathematics who are commonly and naively supposed to acquire an understanding of the reasonableness and relevance of mathematics by osmosis during courses devoted largely to memorized and regurgitated techniques."

Chapter 1 is an introduction to the book. Here are expressed the aims and program of the book and advice to the student. Chapter 2 is devoted to Logic, Mathematics, and Science. Chapters 3 and 4 concern the study of numbers. Chapter 5 is devoted to the logic of algebra. Chapter 6 is titled "Arithmetical Computation and Its Relief." A few pages are devoted to electronic computers and information theory. In Chapter 7 the algebra of logic is discussed. Chapter 8 is called "Impossibilities and Unsolved Problems." In Chapter 9 there are presented the ideas of analytic geometry. Chapter 10 is titled "Functions." In Chapter 11 a discussion of limits and the Calculus is presented. Chapter 12 is devoted to trigonometric functions. The subject of Chapter 13 is probability and statistics. The next chapter is devoted to mathematical induction. Chapter 15 concerns the subject of cardinal numbers. The next chapter discusses Euclidean and Non-Euclidean geometry. The two final chapters cover ideas connected with some simple mathematical sciences and the nature of mathematics.

Excellent bibliographical references are found at the end of each chapter. Those which are of a more advanced or technical nature are suitably marked. Each chapter contains some challenging exercises; the answers to the odd numbered excercises are indicated in the back of the book.

The author has indicated that the chapters are so written that their order of presentation during the course may be arranged to suit the desires of the instructor. He has also indicated that "A student of this text should be able to go on with a sophomore course in analytic geometry and calculus with a little supplementary material, largely analytic trigonometry, which is readily available elsewhere and best discussed when needed."

The book seems to be well written. It appears to be an excellent text book for college freshman in social sciences and other sciences. Also it can give a deeper understanding of mathematics and much pleasure to persons outside the classroom.

(Reviewed by Marvin I. Rosenberg, Commander, US Navy)

EFFICIENCY IN GOVERNMENT THROUGH SYSTEMS ANALYSIS. By Roland N. McKean. New York, John Wiley and Sons, Inc., 1958, 336 + viii pp.

The question of proper allocation of funds for water resources development has, rather surprisingly, been the subject of a considerable amount of debate among economists. To this subject Dr. McKean has brought a great deal of insight and material, but it would be unfair to judge this book on its specific applications alone, for it has important ramifications for an entire field of great importance - the applications of the techniques of operations research and systems analysis to the whole question of governmental budgetary allocation.

The study is divided into five sections. After a brief introduction, Dr. McKean considers the general problems of analysis which are encountered in project evaluation. The first of these involves the question of the choice of appropriate criteria - measures of "desirability" of the various alternatives available. The points covered by McKean in this context include the necessity of using approximate measures in many situations, the "sub-optimizing" problem which arises when a problem is split up into several parts, and the faults of several performance criteria which have been used in such situations in the past. The necessity for choosing the relevant alternatives for comparison in each case, and for allowing for inter-relationships between these alternatives is considered in the second chapter of the section. The third chapter of the section deals with the important questions of how one should deal with intangibles and uncertainty of outcome when these are encountered in evaluation problems. Dr. McKean makes the important point that, while in some situations the pricing of intangibles, as for example, human life, may be impossible, at least one should present the decision maker with budget estimates which show him the monetary costs which must be incurred to obtain intangible gains. On the question of uncertainty, the value of making several estimates based on alternative outcomes, rather than one single estimate, is suggested. These, as well as other suggestions, are in line with the general approach taken by McKean that, while certain decisions must be taken by someone on the basis of experience, it is the duty of the analysist to supply him with as much quantitative information as possible to aid him in such decisions. Finally, the choice of proper discount rates for the present evaluation of projects which will yield a stream of future returns is discussed.

In the next two sections Dr. McKean turns to a consideration of these problems in the context of the general field of evaluation of water resource project evaluation, and then to an application of systems analysis to two specific water resource development projects. Such questions as the usefulness and limitations of the benefits-costs ratio, the proper treatment of external economies resulting from the institution of certain projects, the monetary evaluation

of returns are considered, and suggestions regarding present general practices, and specific procedure in the projects considered are made.

The general reader, whose interests may lie outside the field of resource project evaluation, may find the final section of the book of greatest interest. Chapter 13 deals with the equation of improving performance budgets, performance indicators, and priority listings of projects. In Chapter 14 Dr. McKean examines the question of the extent to which analytical methods might be used to improve the efficiency of governmental operations. On a very rough basis he estimates that 60 percent of non-defense expenditure goes into programs for which meaningful quantitative indicators of performance which are suitable for use in analysis of operations exist or could be derived. Additional opportunities exist for the application of these methods in the defense sector of the budget.

Dr. McKean's study may be considered as an important introduction to both the theory and application of this type of approach in governmental operations. It is clear that a great deal of work remains to be done, both in the general methodology of this sort of decision problem, and on the application of the methods of scientific analysis to other concrete problems of government decision making.

(Reviewed by W. G. Mellon)

INTRODUCTION TO DIFFERENCE EQUATIONS. By Samuel Goldberg. John Wiley and Sons, Inc., New York, 1958. 260 + xii pp. $6.75.

This is a carefully written, lucid, and rigorous treatment of linear difference equations with constant coefficients. The author's objective is to provide a mathematics text for "...social scientists who wish to understand the basic ideas and techniques involved in setting up and solving difference equations." Through the book, difference equation models from psychology, sociology, and economics are used in frequent illustrative examples and exercises; however, no attempt is made to appraise these models on non-mathematical grounds. Rather, this material is used by the author much as problems and examples from mechanics are used in a textbook on elementary calculus; understanding the mathematical development is of primary concern.

The mathematically rigorous nature of the presentation deserves comment. The concepts of set and domain are constantly impressed upon the reader. Consider, for example, the following definition: "An equation relating the values of a function y and one or more of its differences Δy, $\Delta^2 y$, ... for each x-value of some set of numbers S (for which each of these functions is defined) is called a difference equation over the set S." (Page 50.)

In format, the book consists of an introduction and four chapters. The introduction informally defines the difference equation process, with emphasis upon how difference equations have been applied in the social sciences.

Chapter 1 introduces concepts and theorems from the calculus of finite differences necessary as background to difference equation analysis. Operator methods are made familiar, and their usefulness is made clear.

The study of difference equations begins in Chapter 2. A difference equation is defined; the existence of a unique solution for a linear difference equation is proven. Solutions are shown to be sequences; patterns of convergence, divergence, oscillation, and monotonicity are seen as dependent upon certain properties of the sequence.

Chapter 3 deals with methods for solving linear systems with constant coefficients. Conditions for existence of unique solutions of systems of simultaneous linear equations are analyzed. Methods for finding the general solution of the reduced equation and a particular solution of the complete equation are explained. A linear combination of these two solutions is proved to be a general solution of the complete equation.

Chapter 4 treats "selected topics." Perhaps the most interesting of these mathematically is the presentation of a method of solving the difference equation by use of generating function transformations. Generating functions are useful in the analysis of many interesting sequential problems, including probability theory; this exposition should prove valuable in mastery of the method by clarifying it at a rudimentary level.

A characteristic value problem, involving the solution of a difference equation subject to boundary conditions, is considered. Another section outlines the use of matrices when systems of simultaneous linear difference equations arise. Conditions of equilibrium and stability in a second order system are also investigated.

Throughout the book, the author outlines similarities of the calculus of finite differences to ordinary calculus; with emphasis on analogies of difference equation results to those of differential equations.

Difference equation analysis has yielded significant results when applied in the social sciences, as attested to by the examples found in this book; the development of operations or systems research and the growth of use of numerical methods through computer application suggest that more widespread use of the methods of numerical calculus is likely for the future. Finite difference analysis has been used in the production and employment scheduling analysis of Holt, Modigliani, and Simon (Management Science, vol. 2, 1955); the success of their work encourages future applications. A research worker beginning his study of these topics is advised that Introduction to Difference Equations clearly presents basic material, and should serve as an excellent background before a more intensive exploration in the advanced literature and of more complex topics (e.g., non-linear systems, or systems with non-constant coefficients) is attempted. Inclusion of numerous problems from social science models is a boon to the reader interested in these topics, and they greatly facilitate the process of working through the text. Answers are supplied for most of the problems. The text is well documented, with frequent references to primary sources or mathematically more advanced treatments. In all, this readable, useful and unique presentation should find an interested readership among students and workers in the social sciences and applied mathematics.

(Reviewed by Daniel Orr)

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